NUMERICAL EXAMPLES

IN

PHYSICS

(For Intermediate Students.)

PART II

(*Magnetism, Electricity and Sound.*)

Containing 84 solved and 376 unsolved problems.

BY

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FOREWORD.

The book "Numerical Examples in Physics" by Messrs. A. C. Roy B. Sc. and B. P. Srivastava M. A., B. Sc. should meet a long felt want of every teacher of Physics in the Intermediate classes. In this the authors have attempted to frame a number of instructive numerical examples, numbering altogether 723, which they have properly graded for use of a teacher teaching Physics. Though there are a large number of good Text Books on Physics, which contain some very instructive problems there has been great lack of any book that gives them in a collected form. The number of questions is so large and the problems are so varied in their nature that even a teacher, who prefers to set his own problems, will find this book a useful equipment for ready reference. The book ought to be very useful for students too. There is nothing like testing ones knowledge of a subject than by tackling independently numerical problems on it.

The present book attempts to supplement the work of the teacher by adding very lucidly text explanations of the subject matter at the beginning of each chapter, by arousing the curiosity of the student with explana-

tory or informative tables, and by the addition of copious solved problems scattered throughout the book. The number of solved examples in the first part of the book is 84 and of unsolved 179. In the second part of the book dealing with Sound, Electricity and Magnetism the authors have added a considerably larger number of questions, taken from various Indian and Foreign Universities and framed by themselves. The number of solved questions is 84 and of unsolved 376. The first part has been divided into 12 chapters with an appendix and the second part into 7 chapters with an appendix dealing with the use of log tables.

The problems set in this book closely follow the syllabus prescribed by the Boards of High School and Intermediate Education U. P. and Ajemere, C. I. and Rajputana as well as by the Nagpur, Benares and Aligarh Universities for the Intermediate classes. In these days of large classes much of the time of the teacher will be saved if he asked every student of his to have this book with him, and then instead of having to dictate questions every day to his pupils he would only assign the problems that he wants them to tackle.

Messrs. Roy and Srivastava have thus done a real service both to the teachers of Intermediate classes and the student community by bringing out in

this collected form their Numerical Examples in Physics. Their long experience of Intermediate classes entitles their book to be safely put into the hands of the students and to fullest cosideration by the members of the teaching profession.

Ewing Christian College,
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15th. August, 1935.

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PREFACE.

Of all the treatises on problems in Physics that exist in the market, there is hardly any that can claim to cover the just requirements for which it is intended. The students do not profit much from these books when working independently and the teachers too are at a loss to find from any single volume the types of questions which they want to set for their classes.

In view of the above-mentioned difficulties, which confront both the teacher and the taught, the authors of this book, who have been in charge of large groups of students for a number of years, are placing the results of their experience in the form of a book of problems in Physics This small book has been written to meet the requirements of the Intermediate students of these and neighbouring provinces and contains in each chapter a number of solved examples of various kinds followed by a number of exercises. These are properly graded in the order of increasing difficulty.

At the beginning of each chapter a brief, but lucid, text has been incorporated as an aid to the solution of the problems in the chapter. The matter has been so arranged that the student can, by going through the worked out examples, successfully tackle all those that

follow. It is hoped that a student who goes through all the problems will have a good understanding of the subject. At the end the book contains a number of mathematical and conversion tables as appendices, which ought to prove very useful in solving problems. Many of the solved problems have been particularly illustrated with suitable diagrams, and it is expected these will serve as models for the student to form the admirable habit of drawing diagrams to every problem before attempting to solve it. The authors shall consider their labours amply rewarded if the students for whom the book is intended find it useful.

The book has been divided into two parts, the first dealing with Mechanics, Heat and Light and containing 263 problems of various kinds framed by the authors.

The second part deals with Magnetism, Electricity and Sound. This is much bigger in size, the total number of questions being 460 being originally framed by authors or taken from a number of Indian and Foreign Universities. The teacher has thus a much greater choice of types. All the questions are on the course prescribed for Intermediate Classes in the United Provinces, Rajputana and Central India, Central Provinces and other neighbouring Universities. It is, therefore, hoped that in all Colleges of the above

mentioned places the book will be of use both to the students and the teachers.

The authors will be greatly obliged for any suggestions for improvements or corrections where mistakes have crept in the book. They express their indebtedness to Dr. D. S. Kothari M. Sc., Ph. D. (Cantab.) for his opinion and to Professors R. K. Sharma M. Sc. (Head of the Physics Department, E. C. College) and P. K. Sur, M. Sc., Lecturer in Physics, E. C. College, for the kind help they have rendered in seeing the book through the press and in making many valuable suggestions.

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CONTENTS.

CHAPTER I.

MAGNETISM.

Unit pole:—It is that pole which, placed in air, one centimetre from an equal pole repels or attracts it with a force of one dyne. It has no specific name but it is referred to as C. G. S. unit pole.

Laws of attraction or repulsion:—The force between two magnetic poles is directly proportional to the product of the pole strengths and is inversely proportional to the square of the distance apart.

Expressed symbolically,

$$\text{force (F)} = \frac{1}{\mu} \frac{m_1 \times m_2}{d^2} \text{ (dynes)}.$$

where m_1 and m_2 are the pole strengths, and d is the distance apart and μ is the permeability of the medium. (In the case of air the permeability is unity.)

Magnetic field:—The space surrounding a magnet in which magnetic properties can be detected is called the magnetic field. The intensity of the field at any point is measured by the force in dynes experienced by a unit north pole* placed at that point.

*The presence of the north pole is supposed to produce no disturbance in the field.

If the intensity of the field is the same everywhere, the field is said to be uniform. (*e.g.* the earth's field at any place is uniform).

Unit magnetic field :—It is that field which exerts a force of one dyne on a unit north pole placed in it. The name of **Gauss** has been given to this field.

If a magnet of pole strength m is placed in a field of H **Gausses,** then the force exerted on the magnet will be mH dynes.

Magnetic Moment :—It is the moment of the couple required to hold the magnet at right angles to the direction of a field of unit intensity. It is numerically equal to the product of pole strength and the distance between the poles.

Unit :—dynes × cm.

Field due to a bar magnet :—

(*a*) at a point on the axis of a magnet produced is given by the formula.

$$F = \frac{4mdl}{(d^2 - l^2)^2} \text{ (End-on position).}$$

(*b*) at a point on the line bisecting the magnetic axis at right angles is given by the formula.

$$F = \frac{2ml}{(d^2 + l^2)^{\frac{3}{2}}} \text{ (Broad-side on position)}$$

where m represents pole strength of the magnet,

l—half the length of the magnet and *d—the distance of the point from the centre of the magnet.*

If M represents the magnetic moment of the magnet then the two formulæ become

$$(a)\quad F = \frac{2Md}{(d^2-l^2)^2}$$

$$= \frac{2M}{d^3} \text{ if } l \text{ is negligible as compared to } d.$$

$$(b)\quad F = \frac{M}{(d^2+l^2)^{\frac{3}{2}}}$$

$$= \frac{M}{d^3} \text{ if } l \text{ is neglected.}$$

(Action of a magnet in two magnetic fields at right angles. The "A" and "B" Tangent Positions of Gauss.)

$$\text{Case 1.}\quad \frac{M}{H} = \frac{(d^2-l^2)^2}{2d} \tan\theta.$$

$$\text{Case 2.}\quad \frac{M}{H} = (d^2+l^2)^{\frac{3}{2}} \tan\theta$$

(where H is the earth's horizontal field).

See Willow's text book of physics pp. 325—328.

Period of a Vibrating magnet :—If a freely suspended magnet vibrates in a horizonal plane with a

small amplitude in a uniform magnetic field, the time (T) of one oscillation is given by the formula

$$T = 2\pi\sqrt{\frac{K}{MH}}$$

where K—moment of inertia of the magnet

M—magnetic moment

H—horizontal intensity of the field.

Note.—**Moment of inertia :**—If the mass of every particle of a body be multiplied by the square of the distance from the axis of rotation the sum of these products is the moment of inertia of the body about that axis.

For a rectangular bar magnet of mass m, length a, breadth b vibrating about a central axis perpendicular to the axis of the magnet, we have

$$K = m\left(\frac{a^2+b^2}{12}\right)$$

For a cylindrical bar magnet of mass m, length a, and radius r vibrating under similar conditions, we have

$$K = m\left(\frac{a^2}{12}+\frac{r^2}{4}\right)$$

Solved Examples.

1. Find out the nature and magnitude of the force between two similar poles of strength 30 and 60 units situated at a distance of 5 cm. from one another.

Since the poles are similar the force is a repelling one.

$$F = \frac{m_1 \times m_2}{d^2}$$

$$= \frac{30 \times 60}{5 \times 5}$$

$$= 72 \text{ dynes.}$$

2. Two poles, one of which is eight times as strong as the other, exert on each other a force equal to the weight of 500 mi ligrams when placed 10 cm. apart. Find the strength of each. (Inter Board U. P. 1935).

If m represents the strength of the weaker pole, $8m$ will be the strength of the stronger one.

Force between the two poles is given by

$$F = \frac{m \times 8m}{d^2} = \frac{8m^2}{10^2}.$$

(The force here is represented in dynes and the distance in cm.)

Now $\quad$ 1 gram weight = 981 dynes

$$\therefore \quad 500 \text{ milligram} = \frac{981}{2} \text{dynes.}$$

Hence

$$F = \frac{8m^2}{10^2} = \frac{981}{2}.$$

$$\text{Whence } m = \frac{5}{2}\sqrt{981} = 78{\cdot}3.$$

Thus the required pole strengths are 78·3 and 626·4 C. G. S. units respectively.

3. A magnet 15 cm. in length lies in a horizontal uniform magnetic field of intensity 0·36 unit and the strength of each of its poles is 10. Find the moment of the couple required to deflect it through an angle of 30° from the magnetic meridian.

The force acting on each pole is mH dynes $= 10 \times 0{\cdot}36 = 3{\cdot}6$ dynes.

Perpendicular distance between the forces acting on the two poles is $l \sin \theta$ where l is the length of the magnet and θ is the deflection.

$$l \sin \theta = 15 \times \sin 30.$$

$$= 15 \times {\cdot}5 = 7{\cdot}5 \text{ cms.}$$

Hence the required moment of the couple

$= m \text{ H } l \sin \theta$

$= 10 \times 0{\cdot}36 \times 7{\cdot}5$

$= 27$ dynes $\times$ cm or C. G. S. units.

4. A bar magnet is suspended horizontally by a wire. On turning the upper end of the wire half round the magnet is deflected 30° from the meridian. How much the upper end of the wire must be twisted to deflect the magnet 90°?

When the magnet is deflected through an angle θ it is in equilibrium under the action of two opposing couples.

(1) a restoring couple due to the earth's field whose moment is $M H \text{ Sin } \theta$;

(2) a deflecting couple due to the torsion in the wire whose moment is within certain limits proportional to the angle of torsion.

Angle of torsion $\propto MH \sin 30^\circ$.

Here the actual twist in the wire $= 180^\circ - 30^\circ = 150^\circ$ since the magnet is deflected in the same direction as the upper end through 30°.

$\therefore\ 150^\circ \propto M H \sin 30^\circ$. .. .. .. (1)

Let β be the required angle when the magnet is deflected through 90°

Then $\beta - 90^\circ \propto M H \sin 90^\circ$ .. .. (2)

Dividing (2) by (1) we get.

$$\frac{\beta - 90}{150} = \frac{M H \sin 90}{M H \sin 30}$$

$$= \frac{\sin 90^\circ}{\sin 30^\circ} = \frac{1}{\frac{1}{2}} = 2$$

whence $\beta = 390^\circ$

∴ The upper end of the wire should be twisted through 390°.

5. A bar magnet 30 cm. long is placed in the magnetic meridian, and it is found that a small compass needle, placed on the axis of the magnet produced, at a distance of 30 cm. from one pole, will point in any direction. If the strength of the earth's horizontal field is 0·36 C. G. S. unit; what is the pole strength of the magnet? [Inter. Board. 1934]

There are two forces acting on the compass needle —one due to the earth's field and the other due to the magnet. Since at the point in question the compass needle takes any direction it indicates that there is no force acting on it. The two forces then must be balancing each other at this point. (This point is known as the *neutral* point).

Now the field at any point on the axis of a magnet produced is given by the formula.

$$F = \frac{4mdl}{(d^2-l^2)^2}.$$

Substituting the given values,

$$\text{we get } F = \frac{4m \times 45 \times 15}{(45^2-15^2)^2} = \frac{m}{1200}$$

This force must be equal to H *i.e.* 0·36. C. G. S. units.

Hence.

$$F = \frac{m}{1200} = \cdot 36$$

Whence $m = 432$ C. G. S. units.

6. A bar magnet whose poles are 10 cm. apart is placed in the meridian with its north seeking pole to the north. A neutral point is found 10 cm. due east of the centre of the magnet. Calculate the moment of the magnet, given that H = 0·35 C. G. S. units. (Inter. Board 1925)

The field at a point on the line bisecting the magnetic axis at right angles is given by the formula.

$$F = \frac{M}{(d^2+l^2)^{\frac{3}{2}}}$$

Substituting the given values we get.

$$F = \frac{M}{(10^2+5^2)^{\frac{3}{2}}} = \frac{M}{(5\sqrt{5})^3}$$

Since the point in question is a neutral point, the field due to the magnet is equal to the earth's horizontal component H.

$$\text{Hence} \quad F = \frac{M}{(5\sqrt{5})^3} = H = 0\,35$$

Whence M = 489·13 C. G. S. units.

7. A compass needle is placed 30 cms. to the east of a small magnet. The needle is deflected through

45°. Calculate the moment of the magnet approximately and the pole strength if the length of the magnet is 6 cm.

The value of H may be taken as 0·352 Gauss.

(Inter. Board 1932).

From the nature of the question it is evident that the magnet is kept in Tangent A position and hence the field due to the magnet at the point where the compass needle is kept is given by

$$F = \frac{2M}{d^3}$$ (l has been neglected here since the approximate value of M is required).

$$i.e.\ F = \frac{2M}{30^3}$$

Also $F = H \tan\theta$. (See Willow's Text book of Physics pp. 319. and 327.)

$$\therefore \frac{2M}{30^3} = H \tan\theta$$

$$= 0·352 \tan 45^0$$

$$= 0·352 \text{ (since } \tan 45^0 = 1)$$

Whence M = 4752 C. G. S units,

and $m = \frac{4752}{6} = 792$ C. G. S. units.

8. A magnetic needle makes one complete oscillation in 4 secs. in Allahabad where the value of H is 0·35. What will be the value of H at a place where the same needle makes one oscillation in 3 secs ?

(Inter Board 1927)

We know that $T_1 = 2\pi\sqrt{\frac{K}{MH_1}}$

or $T_1^2 = 4\pi^2 \frac{K}{MH_1}$ .. .. .. (1)

Similarly $T_2^2 = 4\pi^2 \frac{K}{MH_2}$ .. .. .. (2)

Dividing (1) by (2) we get

$$\frac{T_1^2}{T_2^2} = \frac{H_2}{H_1} \quad .. \quad .. \quad ... \quad .. \quad (3)$$

Substituting the given values in the formula (3) we get.

$$\frac{4^2}{3^2} = \frac{H_2}{\cdot 35}$$

$$\text{or} \quad \frac{16}{9} = \frac{H_2}{\cdot 35}$$

$$\therefore \quad H_2 = \cdot 35 \times \frac{16}{9}$$

$$= 0{\cdot}62$$

$\therefore$ The required field is 0·62 Gauss.

9. Two magnets are held rigidly one above the other with their axes parallel to each other and similar poles together but without contact. They are suspended and allowed to oscillate in the earth's horizontal field making 30 vibrations per minute. When one of the magnets is reversed it is found that it makes 6 vibrations per minute If the magnetic moment of the stronger magnet is 208, find that of the other.

Let M_1 and M_2 be the moments of the magnets and let t_1 and t_2 be the times of vibrations in the two cases.

The moment of inertia of the whole system will remain the same in both the cases and we shall have

$$t_1 = 2\pi\sqrt{\frac{K}{(M_1+M_2)H}} \quad .. \quad .. \quad (1)$$

$$t_2 = 2\pi\sqrt{\frac{K}{(M_1-M_2)H}} \quad \text{(Where } M_1 > M_2\text{)} \ldots (2)$$

Dividing (1) by (2) we get

$$\frac{t_1}{t_2} = \sqrt{\frac{M_1-M_2}{M_1+M_2}}$$

$$\text{or} \quad \frac{M_1-M_2}{M_1+M_2} = \frac{t_1^2}{t_2^2} \quad \text{(by squaring)}$$

By applying componendo and dividendo we get

$$\frac{M_1}{M_2} = \frac{t_2^2+t_1^2}{t_2^2-t_1^2} \quad .. \quad .. \quad .. \quad .. \quad (3)$$

30 vibrations per min. = 1 vibration in 2 seconds. and 6 vibrations per min. = 1 vibration in 10 seconds.

Substituting these values in the above equation—(3) we get.

$$\frac{M_1}{M_2} = \frac{10^2+2^2}{10^2-2^2}$$

$$= \frac{100+4}{100-4} = \frac{104}{96} = \frac{13}{12}$$

$$\text{or } \frac{208}{M_2} = \frac{13}{12} \text{ for } M_1 \text{ is given to be } 208.$$

$$\therefore \quad M_2 = \frac{12 \times 208}{13} = 192.$$

The required moment of the magnet is 192 C. G. S. units.

10. A compass needle makes 30 oscillations per minute at a place where the dip is 45° and 40 oscillations per min. where the dip is 30°. Find the ratio of the earth's total intensities at the two places

If I denotes the earth's total intensity and H the horizontal component and θ the angle of dip, then

$$\frac{H}{I} = \cos\theta$$

$$\text{or } \quad I = \frac{H}{\cos\theta}$$ (See Willow's Physics pp. 337).

Thus at the first place we have

$$I = \frac{H}{\cos. 45^\circ} = \frac{H}{\frac{1}{\sqrt{2}}} = H\sqrt{2}$$

Similarly at the second place.

$$I' = \frac{H'}{\cos. 30^\circ} = \frac{H'}{\frac{\sqrt{3}}{2}} = \frac{2H'}{\sqrt{3}}$$

The horizontal component of the intensity at any place controls the vibration of the needle at that place

Hence $\frac{H}{H'}=\frac{30^2}{40^2}=\frac{9}{16}$

and $\frac{I}{I'}=\frac{H\sqrt{2}}{\frac{2\ H'}{\sqrt{3}}}=\frac{\sqrt{3}H}{\sqrt{2}\ H'}$

$=\frac{\sqrt{3}}{\sqrt{2}}\times\frac{9}{16}\left(\text{Since }\frac{H}{H'}=\frac{9}{16}\right)$

$=\frac{9}{16}\sqrt{1\cdot5}$

11. A uniformly magnetised magnetic needle has its period of vibration equal to 6 secs. The needle is then broken into exact halves. What is the period of vibration of each half ?

We know that $t_1=2\pi\sqrt{\frac{K_1}{M_1H}}$; $t_2=2\pi\sqrt{\frac{K_2}{M_2H}}$

Again $K_1=w_1\left(\frac{l_1^2}{12}+\frac{r_1^2}{4}\right)=\frac{w_1\ l_1^2}{12}$ approx. (since the radius of the needle is negligible)

and $K_2=\frac{w_2\ l_2^2}{12}$

$\therefore\ \frac{K_1}{K_2}=\frac{w_1\ l_1^2}{w_2\ l_2^2}$

We also know that $M_1=ml_1$ and $M_2=ml_2$ (where l_1 and l_2 are the lengths of the magnets).

$$\therefore \frac{M_2}{M_1} = \frac{l_2}{l_1}$$

Hence $\dfrac{t_1}{t_2} = \sqrt{\dfrac{M_2K_1}{M_1K_2}} = \dfrac{\sqrt{w_1\, l_1^2 \times l_2}}{\sqrt{w_2\, l_2^2 \times l_1}} = \dfrac{\sqrt{w_1\, l_1}}{\sqrt{w_2\, l_2}}$

Now $w_1 = 2w_2$ and $l_1 = 2l_2$

$$\therefore \frac{6}{t_2} = \frac{\sqrt{2w_2 \times 2l_2}}{\sqrt{w_2 \times l_2}} = \frac{2}{1}$$

$$\therefore \quad t_2 = 3 \text{ secs.}$$

12. A small suspended magnet makes 10 oscillations per minute under the influence of the earth's field alone A bar magnet is brought near it so as not to disturb the direction of the pointing of the suspended magnet, but so that the latter now makes 14 oscillations per minute. What would be the frequency if the bar magnet were now reversed pole for pole?

(London Univ. Inter. B. Sc. 1900).

We know that $T = 2\pi \sqrt{\dfrac{K}{MH}}$

Substituting the given values we have

$$\frac{60}{10} = 2\pi \sqrt{\frac{K}{MH}} \quad \ldots\ldots\ldots (1)$$

Let F be the field due to the magnet.

Since the position of the compass needle is not changed by bringing the bar magnet near it, it is

clear that the field due to the magnet is parallel to that of the earth. The compass needle is thus oscillating in the second case under the joint influence of the two fields *i.e.* F+H.

So we have

$$\frac{60}{14} = 2\pi\sqrt{\frac{K}{M\,(F+H)}} \quad \ldots\ldots\ldots \quad (2)$$

From (1) and (2) we get

$$\frac{F+H}{H} = \frac{14^2}{10^2}$$

$$\text{or} \quad \frac{F}{H} + 1 = \frac{49}{25} \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (3)$$

$$\text{or} \quad \frac{F}{H} = \frac{49}{25} - 1$$

$$\therefore \quad = \frac{24}{25}$$

$$\text{or} \quad 1 - \frac{F}{H} = 1 - \frac{24}{25}$$

$$\therefore \quad = \frac{1}{25}$$

$$\text{or} \quad \frac{H-F}{H} = \frac{1}{25} \quad \ldots \quad \ldots \quad \ldots \quad (4)$$

In the third case the compass needle is oscillating in the influence of the difference of the two fields *i.e.* H−F.

We shall have $T_3 = 2\pi\sqrt{\dfrac{K}{M(H-F)}}$ ......(5)

From the equations (1) and (5) we get

$$\frac{T_1^2}{T_3^2} = \frac{H-F}{H}$$

$$\text{or} \quad \frac{6^2}{T_3^2} = \frac{H-F}{H}$$

$$= \frac{1}{25} \quad \text{from equation..(4)}$$

$$\text{or} \quad T_3^2 = 6^2 \times 25$$

$$= 36 \times 25$$

$$\therefore \quad T_3 = 30$$

i e. the time taken for one oscillation is 30 secs.

Hence the compass needle makes 2 vibrations per minute.

13. A small compass needle makes 15 oscillations per min. under the influence of the earth's field. If a long bar magnet be placed vertically with its north pole on the same level with the needle and 30 cm. due south of it, the needle is found to make 20 oscillations per minute. Calculate the pole strength of the magnet, given H = 0·36 C. G. S. unit.

In the first case when the needle is vibrating in the earth's horizontal field we have

$$\frac{60}{15} = 2\pi\sqrt{\frac{K}{MH}} \quad .. \quad ... \quad .. \quad (1)$$

In the second case when the needle is vibrating under the combined effect of the earth's field (H) and the magnet (F), we get

$$\frac{60}{20} = 2\pi\sqrt{\frac{K}{M(H+F)}} \quad \ldots \quad \ldots \quad (2)$$

(The influence of the upper pole is neglected).

From (1) and (2) we get

$$\frac{20^2}{15^2} = \frac{H+F}{H}$$

$$\text{or} \quad \frac{16}{9} = \frac{H+F}{H}$$

$$= 1 + \frac{F}{H}$$

$$\text{or} \quad \frac{F}{H} = \frac{16}{9} - 1 = \frac{7}{9}$$

$$\therefore \quad F = H \times \frac{7}{9} = \cdot 36 \times \frac{7}{9}$$

$$= \cdot 28$$

$$\text{But } F = \frac{m \times 1}{d^2}$$

$$= \frac{m}{30^2}$$

$$i.e. \quad \cdot 28 = \frac{m}{30^2}$$

$$\therefore \quad m = 900 \times \cdot 28$$

$$= 252 \text{ C. G. S units.}$$

14. A bar magnet whose moment is 9360 C. G S. units is turned in a horizontal plane through 60° from the meridian. Find the work done and the couple required to maintain it in that position.

[H = 0·2 dyne per unit pole] [U. L. Inter. 1910]

In fig. 1 let a magnet NS be deflected from the magnetic meridian through an angle θ by a couple.

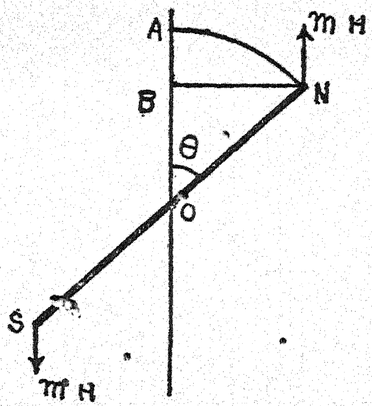

Fig. 1.

The N. pole of the magnet moves along the arc AN but its displacement along the direction of the field is only AB. Hence work done on the north pole = force × displacement = mH × AB.

Similarly an equal amount of work is done on the south pole. Hence if $2l$ represents the length of the magnet, the total work done on the magnet by the couple.

$$= 2\,mH \times AB$$
$$= 2\,mH\,(OA - OB)$$
$$= 2\,mH\,(l - l\cos\theta)$$

(Since OB = $l\cos\theta$]

$$= \quad 2mlH(1-\cos\theta)$$

$$= \quad MH(1-\cos\theta) \text{ ergs} \ldots\ldots (1).$$

Where M is the moment of the magnet.

Substituting the given values in (1) we get,

$$\text{work done} = 9860 \times 0{\cdot}2\,(1-\cos 60)$$

$$= 9860 \times 0{\cdot}2\,(1-\tfrac{1}{2}) \text{ ergs.}$$

$$= 9860 \times 0{\cdot}2 \times 0{\cdot}5$$

$$= 986 \text{ ergs.}$$

And the moment of the couple required to maintain the magnet in that position

$$= \quad MH\sin\theta$$

$$= \quad 9860 \times 0{\cdot}2 \times \sin.\ 60^\circ$$

$$= 986 \times \sqrt{3} \text{ dyne-cm.}$$

Mrs J. Vargis Wesley.

34. A bar magnet is suspended by a wire (without twist) in the magnetic meridian. In order to deflect the magnet 90° from the magnetic meridian the upper end of the wire has to be turned through 230°. Through what angle the upper end of the wire must be turned to deflect the magnet 30° from the meridian ? [Ans. 100°].

35. The moment of a magnet is 1000. How much work is done in turning it through 90° from the meridian at a place where H—0·16 ? [Inter. B. Sc.]

[Ans. 160 ergs.].

36. A magnet of magnetic moment 100 is placed perpendicular to an independent magnetic field of strength 0·5 dyne per unit pole. What couple is required to keep the magnet in this position ? What work would be done by the magnet if it were allowed to turn until its axis became parallel to the direction of the independent field ? [U. L. Inter. 1913.]

[Ans. 50 dyne-cm ; 50 ergs].

37. A bar magnet of magnetic length 10 cm. gives a null point 20 cm. from its centre when placed horizontally in the magnetic meridian with its north pole pointing south. If H = 0·18 gauss, find the moment of the magnet. Where is the null point if the magnet is turned through 180° in a horizontal plane ? (U. L. Inter 1930.)

[Ans. 632·7 C. G. S units ; 14·35 cm from the magnet on line bisecting it at right angles.]

38. There is found to be a neutral point on the prolongation of the axis of a bar magnet at a distance of 10 cm. from the nearest pole. If the length of the bar magnet be 10 cm. and $H = 0·18$ C. G. S. unit find the pole strength of the magnet.

(Camb. Local Senior 1902). [Ans. 24]

39. A bar magnet whose poles are 10 cms. apart is kept in the meridian with its south pole towards north. A neutral point is found at a distance of 15 cm. from the centre of the magnet on its axis. Calculate the moment of the magnet given $H = ·35$ C. G. S. unit. [Ans. $466\frac{2}{3}$ C. G. S units].

40. A bar magnet hung horizontally by a fine wire lies in the magnetic meridian when the wire is without twist. It is then found that when the top of the wire is twisted through $120°$ the magnet is deflected through $30°$. Through what further angle must the top of the wire be twisted in order to turn the magnet perpendicular to the magnetic meridian ?

(S. K. 1893). [Ans. $150°$ more].

41. A bar magnet is suspended horizontally in the magnetic meridian by a wire without torsion. To deflect the bar $10°$ from the meridian the top of the wire has to be turned through $180°$. The bar is

removed, remagnetised and restored and the top of the wire has now to be turned through 250° to deflect the bar as much as before. Compare the magnetic moments of the bar before and after remagnetisation. (B of E. 1894.)

[Ans. 1 : 1·412 [

42. A bar magnet is suspended by a wire so as to hang horizontally. By how much must the top end of the wire be twisted for the magnet to be deflected 90° from the meridian, when for a deflection of 30° it has to be twisted through 120°?

(B of E. 1900). [Ans. 270].

43. The horizontal component of the earth's magnetic field at London and Allahabad are respectively 0·18 and 0·36. If a needle which makes 50 oscillations per minute at London is brought to Allahabad what will be its rate of oscillation?

[Ans. $50\sqrt{2}$ oscillations per min.]

44. The earth's total intensity at Allahabad is 0·45 and the angle of dip is 36°. At New York the two values are given to be 0·61 and 72° respectively. If a magnet vibrating horizontally at New York makes 40 oscillations per minute, how many oscillations would it make in the same time at Allahabad?

$\cos 36° = \cdot 809$; $\cos 72° = \cdot 309$.

$\left[\text{Ans. } 240\sqrt{\frac{1}{19}}\right]$

45. A bar magnet lies on the floor and a magnetic needle is suspended horizontally above it (at a considerable height). The needle makes 24 vibrations per minute when the north pole of the bar magnet points northwards; and when it points south words the needle makes 42 vibrations per min. Find the number of vibrations per minute if the needle were allowed to vibrate only in the earth's field.

$$T = 2\pi\sqrt{\frac{K}{MH}} \quad \text{or} \quad H = \frac{4\pi^2 K}{MT^2} = \frac{4\pi^2 K}{M} \times \frac{1}{T^2}$$

$$= C\,\frac{1}{T^2} \quad \text{(where C is constant for the same magnet)}$$

$$= C.\, n^2 \quad \left(\text{since number of oscillations } n = \frac{1}{T}\right)$$

In the first case the field is H−F

and in the second it is H+F.

$$\text{Hence}\quad H-F = Cn_1^2 \quad \ldots \quad \ldots \quad \ldots \quad (1)$$

$$H+F = Cn_2^2 \quad \ldots \quad \ldots \quad \ldots \quad (2)$$

From (1) and (2) we get

$H = C(n_1^2+n_2^2)/2 = C \times 1170$ (Substituting the values of n_1 and n_2).

But $H = Cn^2$ ∴ It follows that $n = \sqrt{1170}$

$= 34{\cdot}21.$

46. A bar magnet which can move only in a horizontal plane is caused to vibrate at three different stations A, B and C. At A it makes 20 vibrations in 1 min. 30 secs; at B, 25 vibrations in 1 min. 40 secs.; at C, 20 vibrations in 2 mins. Find three numbers proportional to the forces which act upon the magnet at the three places. (B. E.)

[Ans. 64 ; 81 ; 36].

47. A horizontally suspended magnetic needle makes 13 vibrations per minute when no magnets are near. When a bar magnet is placed north of the needle with its axis in the magnetic meridian and its north pole towards the needle the latter makes 7 vibrations per minute. How many vibrations will it make when the bar is similarly placed at the same distance south of the needle its north pole being again directed towards the needle. (B. E. 1899).

[Ans. 17].

48. A magnetic needle makes a complete vibration in a horizontal plane in 2·5 seconds under the influence of the earth's magnetism only, and when the pole of a long bar-magnet is placed in the magnetic meridian in which the needle lies, and 20 cm. from its centre, a complete vibration is made in 1·5 seconds. Assuming H = 0·18 (C. G. S), and neglecting the

torsion of the fibre by which the needle is suspended, determine the strength of the pole of the bar magnet.

[Inter. Sc. Hons. 1886]. [Ans. 128].

49. A uniformly magnetised magnet makes one complete oscillation in 9 secs. The needle is then broken into exact halves. Find out the period of vibration of each half.

[Ans 4·5 secs.]

50. On a table where the earth's magnetism is counteracted the north pole of a compass needle makes 20 oscillations in a minute under the attraction of a south pole 4 inches distant; how many oscillations will it make when the south pole is 3 inches distant?

(Allahabad University) [Ans. 26·67].

51. Two magnets of moments 3 and 4 units respectively are suspended with their centres in the same vertical line and (*a*) their axes in the same vertical plane with like poles together (*b*) their axes in the vertical plane with unlike poles together (*c*) their axes at right angles. Compare the times of swing of the system in the three cases.

(U. L. Inter. 1929). $\left[\text{Ans. } \frac{1}{\sqrt{7}} : 1 : \frac{1}{\sqrt{5}}\right]$.

Hints—In the above question the moment of inertia and the magnetic moment of the system are obtained by simple addition with regard to the

moments of inertia and by vector addition* with regard to the magnetic moments. Hence the moment of inertia of the system $= K_1 + K_2$ and the moment of the equivalent magnet $= \sqrt{M_1^2 + M_2^2}$.

$$\text{Hence, } T = 2\pi \sqrt{\frac{K_1 + K_2}{(\sqrt{M_1^2 + M_2^2})H}}.$$

52. A solid cylinderical bar magnet of steel 10 cm. long and 1 cm. diameter, vibrates once in 10 secs, in the earth's horizontal field. If the density of steel is 8 grammes per c. cm. and $H = 0{\cdot}18$ C. G. S. unit, calculate the value of the magnetic moment of the magnet. (Inter. B. Sc.)

[Ans. 1157].

53. Two magnets of the same steel, dimensions $10 \times 3 \times 0{\cdot}5$ and $20 \times 4 \times 0{\cdot}7$ respectively were found, when pivoted horizontally in the earth's field so as to oscillate about their shortest axes to swing at the same rate. Compare the magnetic moments and the intensities of magnetisation of the two. (B. Sc.)

$$\left[\text{Ans.} \begin{array}{l} (1)\ 1 : 14{\cdot}2 \\ (2)\ 1 : 3{\cdot}82 \end{array} \right]$$

* The vector addition is the resultant obtained by the law of parallelogram of forces.

End

CHAPTER II

ELECTROSTATICS.

Laws of Attraction and Repulison :—

The force of attraction between two oppositely electrified bodies or the force of repulsion between two similarly electrified bodies is found to depend on three things:—

(1) degree of electrification of each of the bodies,

(2) their distance apart,

and (3) the nature of the intervening medium.

To represent it mathematically, if Q and Q′ denote the charges (strictly point charges), d—the distance apart and F—the force between them, then

$$F = \frac{1}{K} \cdot \frac{QQ'}{d^2}$$

where K is a number called the **Dielectric constant** of the medium whose value depends upon the nature of the surrounding medium and which, for air, is taken as unity.

C. G. S. Electrostatic Units :—

Unit Charge.—The Electrostatic unit of electricity is that quantity which if placed 1 cm in air from an equal and like quantity repels it with a force of 1 dyne. The charges are supposed to be point charges and the medium between them being air.

Note :—The practical unit is the **Coulomb** and it is equal to 3×10^9 electrostatic units.

Electric Field :—The space surrounding a charged body in which electrical properties can be detected is called an electric field.

Strength of the electric field, or Electric Intensity or the Electric force at a point:—It is measured by the force in dynes experienced by a unit positive charge placed at that point.

In terms of lines of force the strength of the field at a point is given to be 4π times the density of lines of force at that point i.e. $F = 4\pi N$ where N represents the density of the lines of force.

Difference of Potential :—Difference of potential between two points is measured by the work done in *ergs* on or by a unit positive charge in moving it from one point to the other. If the work done is 1 *erg* the difference of potential is said to be unit.

Potential at any point :—It is measured by the work done on or by a positive unit charge when it is brought from an infinite distance to that point.

Note :—The practical unit is the **Volt** and it is equal to $\frac{1}{300}$ of an electrostatic unit. But 1 electrostatic unit $= 3 \times 10^{10}$ electromagnetic units; hence 1 **Volt** $= \frac{1}{300} \times 3 \times 10^{10} = 10^8$ e. m. units.

Capacity :—If a conductor be charged with Q quantity of electricity and if V be the potential to which it is raised then the ratio of Q to V is called its Capacity *i.e.* Capacity $= \frac{\text{Quantity}}{\text{Potential}} \left(C = \frac{Q}{V} \right)$

In this equation if V is taken to be unity, then $C = Q$ and hence the capacity of a conductor can also be defined as the quantity of charge required to raise the potential of the conductor by unity

Unit Capacity :—A conductor is said to have unit capacity when unit charge raises its potential from zero to unity.

Note:—The practical unit of capacity is **Farad.** A conductor has a capacity of one Farad if a charge of one Coulomb raises its potential by one Volt.

$$\textbf{FARAD} = 9 \times 10^{11} \; e.\,s. \text{ units.}$$

$$= \frac{1}{10^{9}} \; e.\,m. \text{ units.}$$

(For 1 *e. m.* unit of capacity $= 9 \times 10^{20}$ *e. s.* units.)

The capacity of a conductor in a medium of **Specific Inductive capacity** * K is equal to K times the capacity of the conductor in air.

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* Specific inductive capacity or dielectric constant of a medium is the ratio of the capacity of a conductor with the given medium as the dielectric to its capacity with air as dielectric.

Capacity of a sphere :—It is numerically equal to the radius of the sphere (*i.e.* $C = R$)

Capacity of a spherical condenser.

$$= K \frac{R_1 R_2}{R_2 - R_1} \; e.\,s. \text{ units},$$

where K is dielectric constant,

R_1—radius of inner sphere,

R_2—radius of outer sphere.

Capacity of parallel plate condenser

$$= K.\frac{S}{4\pi d} \quad e,\,s. \text{ units},$$

where K is the dielectric constant,

S—the area of the plate,

and d—distance between the plates.

In case of air as dielectric K is taken to be unity.

See Willow's Text book of Physics pp. 438 and 439.

Solved Examples.

1. Two bodies charged with 21 and 44 units are placed in a medium of dielectric constant 2·31 at a distance of 4 cm. Find the force between the two bodies.

$$\text{We know that } F = \frac{1}{K} \cdot \frac{QQ'}{d^2}$$

$$= \frac{1}{2\cdot31} \times \frac{21 \times 44}{4 \times 4}$$

$$= 25 \text{ dynes.}$$

2. Two conductors of capacities 10 and 15 respectively are connected by a fine wire and a charge of 1000 units is divided between them. Find the potential of each conductor and the charge of each.

(C. U. 1920).

$$\text{We know that } C = \frac{Q}{V}$$

$$\therefore V = \frac{Q}{C}$$

$$= \frac{\text{charge}}{\text{Total capacity}}$$

$$= \frac{1000}{15+10}$$ (since the conductors are connected together, their total capacity $= 15+10=25$)

Hence $V = 40$.

$\therefore$ The charge on the first conductor $= 10 \times 40$
$= 400$ units.

Similarly the charge on the second conductor
$= 15 \times 40 = 600$ units.

3. What charge is required to electrify a sphere of 20 cm radius until the surface density of electrification is $\frac{3}{\pi}$?

The surface density is the quantity of electricity per one square cm. of surface. If S be the area of the surface and d—the surface density, then

$$Q = S \times d.$$

The area of the surface of a sphere of radius R is

$$4\pi R^2.$$

$$\therefore \quad S = 4\pi\ (20)^2.$$

$$\text{Hence} \quad Q = 4\pi\ (20)^2 \times \frac{3}{\pi}$$

$$= 4800 \text{ units.}$$

4 A small brass sphere is charged with 50 units of positive electricity and is made to touch another sphere of same radius having a charge of 10 units of negative electricity. Find the force between the two spheres when they are separated by a distance of 20 cms.

What will be the force, with the above conditions, between the spheres if they are immersed in oil of specific inductive capacity of 3 ?

(Inter. Board. U. P. 1932.)

Total charge on both the spheres after they have touched one another = 50 − 10 = 40 positive units.

Since the spheres are of equal radii and their potentials are the same, the charges will distribute equally when they are separated. Hence the charge on each sphere after separation is equal to (40/2) or 20 positive units.

Hence the required force between them is given by

$$F = \frac{QQ'}{d^2}$$

$$= \frac{20 \times 20}{20^2}$$

$$= 1 \text{ dyne.}$$

In the second case

$$F = \frac{1}{K}\frac{QQ'}{d^2}$$

$$= \frac{1}{3}\frac{20 \times 20}{20^2}$$

$$= \frac{1}{3} \text{ dyne.}$$

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5. From a point two small spheres each of mass m gms. are suspended by threads Both are equally charged. If the length of each thread be a cm. long and the distance between the two spheres be b cm; find out the charge on each in (1) electrostatic uuits (2) eletromagnetic units (3) coulombs.

The following three forces act on the sphere at A (Fig 3) :—

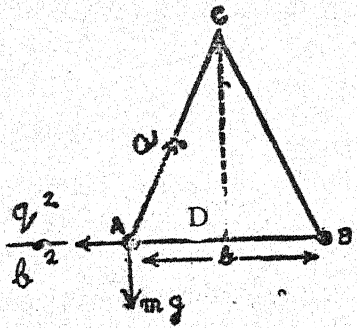

Fig 3.

(1). The weight, mg dynes acting downwards.

(2). The repulsion $\frac{q^2}{b^2}$ dynes acting in the direction of BA.

(3). The tension of the thread acting in the direction of AC.

The triangle CDA has its sides parallel to these forces.

∴ The forces are proportional to the sides of the triangle to which they are parallel

So we have

$$\frac{\frac{q^2}{b^2}}{mg} = \frac{DA}{CD} = \frac{\frac{b}{2}}{\sqrt{a^2 - \frac{b^2}{4}}}$$

$$\therefore \quad q^2 = \frac{mgb^3}{2\sqrt{a^2 - \frac{b^2}{4}}}$$

$$= \frac{mgb^3}{\sqrt{4a^2 - b^2}}$$

Hence $q = b\sqrt{\dfrac{mgb}{\sqrt{4a^2 - b^2}}}$ electrostatic units.

In electromagnetic units the charge is $\dfrac{q}{3 \times 10^{10}}$ and in coulombs $\dfrac{q}{3 \times 10^9}$.

6. A spherical conductor of 8 cm. radius is charged to a potential of 80 units. It is placed inside an uncharged spherical conductor of 16 cm. radius and is allowed to touch its inner surface. To what potential will the larger conductor be raised if the medium is air?

When the smaller sphere touches the larger one, all its charge passes to the latter. Let V be the potential to which the larger sphere is raised due to this charge.

Hence we have

$$8 \times 80 = 16 \times V$$

$$\therefore \quad V = 40 \text{ C. G. S. units.}$$

7. A spherical conductor of 10 cm. radius, charged to a potential of 50 units is discharged by contact with

the inner surface of an other spherical conductor of 20 cm. radius. It is then taken out and connected by a long thin wire with the larg er conductor. Find the common potential and the charge on each.

The potential of the bigger sphere after the smaller one has touched it is given by

$20 \times V = 10 \times 50.$ (See previous example).

$\therefore V = 25$ C. G. S. units.

Let V' be the common potential of the conductors when they are connected by a thin wire.

Then the charge on the inner sphere $= 10 \times V'$ ($\because Q = C.V.$) and C (capacity) in the case of a sphere is equal to its radius. Similarly the charge on the larger sphere $= 20 \times V'$. Hence the total charge on both the spheres $= 10\ V' + 20 V' = V'(10 + 20)$.

After the inner sphere has touched the bigger one all its charge passes to the bigger one. The charge on the smaller sphere becomes zero and that on the larger one $= 25 \times 20$ units. Hence we have,

$V'(10 + 20) = 25 \times 20 + 0$

$\therefore V' = 16\frac{2}{3}$.

Hence the charge on the smaller conductor

$$= \frac{50}{3} \times 10$$

$$= \frac{500}{3} \text{ units}$$

$$= 166\tfrac{2}{3} \text{ units,}$$

and the charge on the bigger conductor

$$= \frac{50}{3} \times 20$$

$$= \frac{1000}{3}$$

$$= 333\tfrac{1}{3} \text{ units.}$$

8. Within a spherical vessel of copper 2 centimetres thick the external diameter of which is 16 cm. a brass ball 10 cm. in diameter is hung by a silk thread in such a way that the centres of the two spheres coincide If the ball is charged with 40 units of positive electricity and if the potential of the vessel is 8, find the potential of the ball.?

The radius of the ball is 5 cm. and the inner and the outer radii of the vessel are respectively 6 cm. and 8 cm.

Since the vessel is spherical its capacity is numerically equal to its radius i.e. $C = 8$.

Now if Q be the charge on the outer surface of the vessel we shall have $Q = C \times V = 8 \times 8$ (since the potential of the vessel is given to be 8) $= 64$ units.

The charge on the ball is 40 units of positive electricity and this will induce a charge of -40 units on the inner side of the vessel.

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The potential of the ball will be equal to the sum of the potentials due to its own charge and the charges on the inner and outer surfaces of the vessel.

Thus if V represents the potential of the ball we have.

$$V = \frac{q_1}{r_1} + \frac{q_2}{r_2} + \frac{q_3}{r_3}$$

$$= \frac{40}{5} + \frac{-40}{6} + \frac{64}{8}$$

$$= 8 - 6\tfrac{2}{3} + 8$$

$$= 9\tfrac{1}{3} \text{ e. s. units.}$$

9. ABCDEF is a regular hexagon of 2 cm. side having charges $+1$, -2, -3, $+4$, $+6$ at its angular points, A, B, C, D and E respectively. Find the potential at F.

AF = 2 cm.; AB = 2 cm.

$$\therefore BF = \sqrt{2^2 + 2^2 + 2.2.2\cos 60}.$$

$$= \sqrt{12}.$$

$$= 2\sqrt{3}.$$

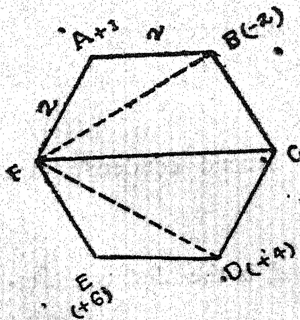

Fig. 4.

CF = 4 cm. (from the property of regular hexagon).

$DF = BF = \sqrt{12} = 2\sqrt{3}$.

Hence if V be the potential at F, we have

$$V = \frac{+1}{2} + \left(\frac{-2}{2\sqrt{3}}\right) + \left(\frac{-3}{4}\right) + \left(\frac{+4}{2\sqrt{3}}\right) + \left(\frac{+6}{2}\right)$$

$$= \tfrac{1}{2} - \frac{1}{\sqrt{3}} - \tfrac{3}{4} + \frac{2}{\sqrt{3}} + 3.$$

$$= \frac{4 + 11\sqrt{3}}{4\sqrt{3}} = \frac{1}{\sqrt{3}} + \frac{11}{4}$$

$$= 2{\cdot}75 + \frac{1}{\sqrt{3}} \quad \text{e. s units.}$$

10. Two condensers have plates of area 100 sq. cms. and 625 sq. cms. respectively and the distance apart of the plates of the smaller one is 1 millimeter. If equal charges are given to the two condensers, find the distance apart of the plates of the second condenser which will cause equal potential differences in the two condensers. (Inter. Board U. P. 1934).

We know that $C = \dfrac{K \times S}{4\pi d}$.

(*a*) In the case of the first condenser,

we shall have $C = \dfrac{100\ K}{4\pi \times {\cdot}1}$.

Similarly in the case of the second condenser we shall have

$C' = \dfrac{625K}{4\pi d'}$ where d' is the distance between the plates.

Since equal charges are given to both the condensers and they are raised to the same potential, their capacities must be the same.

Thus we have

$$C = C'$$

$$\frac{100\ K}{4\pi \times \cdot 1} = \frac{625K}{4\pi d'}$$

whence $d' = 6\cdot 25$ mm.

11. An air condenser has plates of 6 cm. diameter. At what distance should the plates be placed so as to have the same capacity as a sphere of diameter of 100 cm? [Inter. Board U. P. 1935].

We know that the capacity of a sphere is numerically equal to its radius and so the capacity of a sphere of diameter of 100 cm. is equal to 50.

We know that the capacity of parallel plate condenser $= K\frac{S}{4\pi d}$ where K is the dielectric constant,

S – area of the plate,

d – distance between the plates.

In the case of air condenser $K = 1$ and so we have

$$C = \frac{S}{4\pi d}.$$

By the question we have

$$50 = \frac{\pi (3)^2}{4\pi d}$$

$$= \frac{9}{4d}.$$

$$\text{Whence } d = \frac{9}{200} \text{ cm.} = \cdot 045 \text{ cm.}$$

12. A conducting sphere is surrounded by an earthed concentric sphere. If the radii of the spheres are 50 and 50·5 cm, calculate the capacity of the inner sphere.

The two spheres from a spherical air condenser since the outer one is earthed.

The capacity of the inner sphere is given by

$$C = \frac{R_1 R_2}{R_2 - R_1}$$

$$= \frac{50 \times 50 \cdot 5}{(50 \cdot 5 - 50)}$$

$$= 5050 \textit{ e. s.} \text{ units.}$$

13. An uncharged condenser containing a solid dielectric is connected to an equal and similar air condenser which is charged to a potential 100. If the common potential after sharing the charge is 25, calculate the specific inductive capacity of the solid dielectric.

Let C be the capacity of the air condenser, then its original charge must have been $Q = VC = 100 \times C$.

Let C' be the capacity of the uncharged condenser, then the joint capacity of the two condensers after connection $= C + C'$: hence the total charge $= (C + C')\,25$.

Since no electricity is lost in sharing the charge this quantity of electricity must be equal to the original charge.

Hence we have

$$100 \times C = (C + C')\,25 \ldots\ldots (1)$$

But $C' = CK$ if K be the S.I.C. of the solid dielectric.

The equation (1) then becomes.

$$100 \times C = (C + CK)\,25$$

$$\therefore K = 3.$$

14. A spherical conductor A charged to a potential of 400 units shares some of its charge to another spherical conductor B of 10 cm. radius. The potential of A is then found to be 200 units. Find the capacity of A.

After the two conductors have touched each other they will have the same potential and consequently the potential of A is that of B = 200 units.

We know that $Q = C \times V$

$= R \times V$ (where R is the radius)

The charge taken by the conductor B to raise its potential from 0 to 200 is given by

$$Q = C \times V$$

$$= R \times V$$

$$= 10 \times 200$$

$$= 2000 \text{ units.}$$

This charge was given by A and thus its potential fell through 200 units.

If C′ be the capacity of A then we have

$$C' = \frac{Q}{V} = \frac{2000}{200}$$

$$= 10 \text{ C. G. S. units.}$$

15. Three condensers of capacities 10, 20 and 30 e.s. units are connected in series. The insulated plate of the first condenser is at a potential of 121 e s. units. If one plate of the third condenser be earthed, what is the difference of potential between the plates of the second condenser ?

We know that the capacity of condensers connected in series is given by

$$\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}$$

$$= \frac{1}{10} + \frac{1}{20} + \frac{1}{30}$$

$$= \frac{6+3+2}{60}$$

$$= \frac{11}{60}$$

$$\therefore C = \frac{60}{11}$$

We also know that $Q = C \times V$ (where V represents the difference of potential between the first and last plate). Substituting the values we get

$$Q = \frac{60}{11} \times 121 \quad [V = (121 - 0) \text{ since the last plate of the third condenser is earthed}]$$

$$= 660 \text{ units.}$$

This gives the charge on each condenser. Since the capacity of the second condenser is 20, the difference of potential between its plates $= \frac{660}{20} = 33$ e. s. units.

16. A Leclanche cell of E.M.F. 1·4 volts is used in charging a parallel plate condenser of capacity 2 micro-farads. Supposing the charge given to the condenser be used to raise the metal spheres of 30 cms. radius to a potential of 4000 volts, how many such spheres could be charged ?

We know that 1 Volt $= \frac{1}{300}$ e.s. unit.

$$\therefore 1 \cdot 4 \text{ Volts} = \frac{1 \cdot 4}{300} \text{ e.s. unit.}$$

Again 1 Farad $= 9 \times 10^{11}$ e s. units.

$$\therefore 1 \text{ micro-farad} = \frac{9 \times 10^{11}}{10^6} = 9 \times 10^5 \text{ e.s. units.}$$

$$\therefore 2 \text{ micro-farads} = 2 \times 9 \times 10^5 \text{ e.s. units.}$$

The quantity of charge taken by the condenser
$=$ Capacity $\times$ Potential

$$=2\times 9\times 10^{5} \times \frac{1\cdot 4}{300} \text{ units.}$$

$$=8\cdot 4\times 10^{3} \text{ units.}$$

The charge taken by each sphere of 30 cm. radius in order to raise its potential through 4000 volts

$$=30\times\frac{4000}{300} \text{ e. s. units.}$$

$$= 400 \text{ e. s. units.}$$

Hence the required number of spheres $=\dfrac{8\cdot 4\times 10^{3}}{400}$

$$=21.$$

17. A condenser A has plates of area 1000, and a dielectric of thickness 4; another condenser B has plates of area 800 and the same dielectric of thickness 5. Compare the charges and energy in A and B when they are connected, A to a source of potential 4, and B to a source of potential 5. [L.'96]

The capacity C of a condenser is given by

$$C=\frac{KS}{4\pi d}.$$

Hence the capacities of the two condensers are given

by (1) $C_1 = \dfrac{K \times 1000}{4\pi \times 4}$,

(2) $C_2 = \dfrac{K \times 800}{4\pi \times 5}$

$\therefore \dfrac{C_1}{C_2} = \dfrac{25}{16}$.

But we know that $Q_1 = C_1 V_1$

and $Q_2 = C_2 V_2$

$\therefore \dfrac{Q_1}{Q_2} = \dfrac{C_1 V_1}{C_2 V_2} = \dfrac{25}{16} \times \dfrac{4}{5} = \dfrac{5}{4}$.

The energy is given by

$E_1 = \frac{1}{2} Q_1 V_1$

and $E_2 = \frac{1}{2} Q_2 V_2$

Hence $\dfrac{E_1}{E_2} = \dfrac{\frac{1}{2} Q_1 V_1}{\frac{1}{2} Q_2 V_2}$

$= \dfrac{Q_1 V_1}{Q_2 V_2}$

$= \frac{5}{4} \times \frac{4}{5}$

$= 1$.

$\therefore$ The energy will be equal in both.

18. The armatures of a condenser are concentric spheres of radius 8 and 10 cm respectively and the space between them is filled with paraffin wax of S. I. C. = 2. The condenser has a charge of 8000 e. s.

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units. How much heat will be produced if the condenser be discharged in such a way that all its energy is converted into heat ? ($J = 4 \cdot 2 \times 10^7$ ergs)

The capacity of the condenser is given by

$$C = K \frac{R_1 R_2}{R_2 - R_1}$$

$$= 2 \times \frac{8 \times 10}{10 - 8} = 80 \text{ units.}$$

The energy of its charge $= \frac{1}{2} \frac{Q^2}{C}$

[See Willow's text book of physics p. 441].

$$= \frac{1}{2} \frac{(8000)^2}{80}$$

$$= 400000 \text{ ergs.}$$

But $W = JH$.

$\therefore$ Heat produced $= \dfrac{400000}{4 \cdot 2 \times 10^7}$

$$= \frac{1}{105} \text{ calorie.}$$

19. Two spheres of radii 8 and 12 cm. have equal charges of 192 units each. They are then joined by a thin wire so that their charges are re-distributed over them. Calculate the loss of energy.

We know that $Q = C_1 V_1$

$= R_1 \times V_1$ (in the case of a sphere conductor).

$$\therefore V_1 = \frac{Q}{R}$$

$$= \frac{192}{8} = 24 \text{ in the case of first sphere.}$$

Similarly the potential of the other

$$\text{sphere } V_2 = \frac{192}{12} = 16.$$

$$\text{Common potential} = \frac{192+192}{8+12}$$

$$= \frac{96}{5}$$

$$\text{Energy before sharing} = \tfrac{1}{2} C_1V_1^2 + \tfrac{1}{2}C_2V_2^2$$
$$= \tfrac{1}{2} \times 8 \times 24^2 + \tfrac{1}{2} \times 12 \times 16^2$$
$$= 3840 \text{ units.}$$

$$\text{Energy after sharing} = \tfrac{1}{2} \times 20 \times \frac{96^2}{5^2} = 3686{\cdot}4$$

(since common capacity $= 12 + 8 = 20$)

$$\text{Hence loss of energy} = 3840 - 3686{\cdot}4$$
$$= 153{\cdot}6 \text{ units.}$$

The End

Exercises.

Note.—All quantities are expressed in terms of C. G. S. units).

1. The distance between two small electrified bodies is 10 cm. Each is charged with 40 units of positive electricity. Find the force of repulsion between them if the intervening medium is (1) air (2) a medium of dielectric constant 2.

(Ans. (1) 16 dynes. (2) 8 dynes.)

2. Calculate the force between two small bodies charged with +35 and +25 units of electricity placed in a medium of specific inductive capacity 2·5, the distance between them being 5 cm. [Ans. 14 dynes].

3. The distance between two small charged spheres is 8 cm. One of them has a charge of 72 units. Find the charge on the other sphere if the force of attraction between them is equal to a weight of 100 milligrams.

[Ans. 87·2 units].

4. Two similarly charged spheres placed 6 cm. apart in a medium of dielectric constant 3 repel each other with a force of 140 dynes. If one has a charge of 60 units, find the charge on the other.

[Ans. 252 units].

5. What must be the distance between two small insulated spheres having charges of 160 and 40 units if the repulsive force between them be 100 dynes ?

[Ans. 8 cm.]

6. Two similar balls of which one is electrified are placed in contact. On being separated 10 cm. from one another the repulsive force between them is equal to 25 dynes. What was the original charge on the electrified ball ?

[Ans. 100 units].

7. A small insulated sphere A charged with the quantity of electricity 2 is at a distance of 25 ~~mm~~ Cm from a second similar sphere B charged with the quantity 5 ; the latter is momentarily touched with an unelectrified sphere D of the same size, and the distance altered to 20 ~~mm.~~ Cm What is the ratio of the repulsive forces in the two cases ? (A. U.)

[Ans. 32: 25].

8. Electric charges of 10 and 5 units are given to two bodies which are at a distance of 50 cm. apart. At what point on the straight line joining the charges is the electric force zero? [C. of P. Senior 1906]

[Ans. 20·7 cm. from the smaller charge].

9. Two charged conducting spheres repel each other with a force equal to the weight of a milligram when placed at a certain distance from each other.

If the charge on one of the spheres is doubled and the distance between the spheres is also doubled, what is the amount of repulsion ? [B of E. 1933].

[Ans. 0·5 milligram.]

10. Two small balls of equal diameter charged with -15 and $+20$ units of electricity are placed 4 cm. apart. They are made to touch momentarily and then again separated by the same distance. What is the force of attraction and repulsion before and after contact.

$$\left[\text{Ans } 18\tfrac{3}{4} \text{ dynes; } \frac{25}{64} \text{ dyne} \right].$$

11. Two insulated metal balls with radius 6 cm. and 9 cm. are connected by a fine wire. They are charged and the larger is found to have a charge of 30 units. What was the total charge ?

[Ans. 50 units].

12. A sphere of 10 cm. radius, charged with 250 units of electricity is placed in contact with another uncharged sphere of 15 cm. radius. They are then separated and placed 20 cm. apart. Calculate the force between them.

[Ans. 37·5 dynes].

13. A small pith-ball weighing 1 decigram, suspended by a silk fibre and charged with positive electricity, is repelled when a charged glass rod is brought

near it. If the direction of the electric field of the glass rod near the ball is horizontal and its magnitude equal to 20 C. G. S electrostatic units, when the deflection of the fibre is 45°, what is the charge on the ball? (B of E. 1906). [Ans. 4·9].

14. Two small spheres each of mass one decigramme are suspended from a point by threads each 50 cm. long. They are equally charged and they repel each other to a distance of 20 cm. If $g=980$ cm/sec^2 what is the charge on each? (Inter. Board U. P. 1930)

$$\left[\text{Ans. } \frac{140}{(6)^{\frac{1}{4}}}\right]$$

15. Two copper spheres each 1 millimetre in diameter are suspended from the same point by silk fibres 1 metre long, and when equally charged are at a distance of 1 centimetre from centre to centre. Determine the charge on each sphere, the density of copper being 8·9 and the acceleration of gravity 980. (B. of E. 1907) [Ans. 0·151]

16. At the two corners of an equilateral triangle of 15 cm side are placed + 20 and − 20 units. Find the magnitude and direction of the resultant force acting on a charge of +20 units placed at the third corner of the triangle.

[Ans. $1\frac{7}{9}$ dynes].

17. At the three corners A, C, and D of a square ABCD of 5 cm side, charges + 10, − 10 and + 15 electrostatic units are placed. Find the intensity of the field at B, the medium being air.

[Ans. $\frac{1}{10}\sqrt{41}$ e. s. unit].

18. The diameters of three insulated metal balls are respectively 10, 14 and 24 inches. They are connected together by a fine wire. What are the charges on the other balls if the smaller ball has a charge of 20?

[Ans. Second ball, 28 ; third ball, 48].

19. What charge must be given to spherical conductor of 6 cm diameter so that the surface density of the electrification may be 7 ?

(Take $\pi = \frac{22}{7}$) [Ans. 792 units].

20. Two insulated spheres A and B whose diameters are respectively as 7 : 10 have equal quantities of electricity imparted to them. In what ratio are their electric densities.

(A. University) [Ans. 100 : 49].

21. In moving a charge of 100 e. s. units through a distance of 10 cm. in a uniform field 3000 ergs of work are done. Find the intensity of the field and the density of the lines of force.

[Ans. (1) 3 e. s. units ; (2) $\frac{3}{4\pi}$]

22. The potential of a spherical conductor in air is raised from 20 to 30 units by giving it a charge of 80 C. G. S. units. Find the radius of the conductor.

[Ans. 8 cm].

23. Find the quantity of electricity which must be given to an insulated sphere of 12 cm. diameter so that its potential may be raised from zero to 30.

[Ans. 180 e. s. units].

24. A charge of +25 e. s. units moves along a line of force in a uniform field from a point P to a point Q. If the work done is 1800 ergs and if the distance PQ is 2 cm, find the potential difference between P and Q and the intensity of the field.

[Ans. (1) 72 e. s, units. (2) 36 e. s. units.]

25. 50 units of positive electricity are placed at the middle points of the sides of an equilateral triangle, the sides of which are 10 cm long. Find the potential at the centre of the inscribed circle.

[Ans $30\sqrt{3}$ e. s. units].

26. At the corners A, B and C of a square are placed charges of 40, 80 and 100 units of positive electricity. The sides of the square are 20 cm long.

Find the potentials at the corner D and at the centre O. Find the amount of work necessary to be done in order to bring a positive unit from D to O.

$$\left(\text{Ans. } \begin{array}{l} V_d = 9{\cdot}828. \\ V_0 = 15{\cdot}554. \\ \text{Work} = 5{\cdot}726 \text{ ergs.} \end{array} \right)$$

27. Three insulated metal spheres are charged with electricity till their potentials are raised to 9, 12 and 14 respectively. They are placed at considerable distances apart The radii of the spheres are 2, 3 and 5 cm respectively. Find the potential of the whole system when they are connected by a fine wire.

[Ans. 12·4].

28. The diameters of two insulated copper balls are $5''$ and $2''$. They are joined by a long fine wire. If 98 units of electricity are given to them, find how the charge will be distributed? [Ans. 70 and 28 units]

29. An insulated sphere has its radius equal to 10 cm. To what potential should we charge it so that its surface density may be represented by 5?

[Ans. 628·32].

30. Two insulated and widely separated metallic spheres recieve charges of positive electricity which raise their potentials to 4 and 5 respectively. The densities of the charges being in the ratio 4 : 9, compare the radii of the balls. (S K. 1889).

[Ans. 9 : 5].

31. Two insulated brass balls are connected by a long fine wire and are charged to a potential 80. If the diameter of one is twenty times that of the other, compare their densities.

[Ans. 1 : 20].

32. How much energy is expended in carrying a charge of 50 units of electricity from a place where the potential is 20 to another where it is 30? What is meant by saying that the ptoential of a conductor is 20 ? (B of E. 1894.)

[Ans. 500 ergs].

33. At each of the four corners of a square of 8 cm. side are placed four equal charges each of 16 units. Find the potential at the middle point of either of the sides and at the centre of the square.

$$\left[\text{Ans. } 8\sqrt{2};\ 8\left(1+\frac{\sqrt{5}}{5}\right)\right]$$

34. Charges of $+5, +10, -15, +20, -25$, units of electricity are placed at the corners A, B, C, D, and E of a regular hexagon ABCDEF having one of its sides equal to 4 cm. Find out the potential at the point F.

$$\left[\text{Ans.} \left(\frac{5\sqrt{3}}{2}-\frac{55}{8}\right)\text{e.s. units.}\right]$$

35. A conducting sphere of diameter 6, is electrified with 105 units, it is then enclosed concentrically

within an insulated and unelectrified hollow conducting sphere formed of two hemispheres of thickness $\frac{1}{2}$ and internal diameter 7. The outer sphere is then put to earth. Determine the potential of the inner sphere before and after the outer sphere is earth connected. (B. of E. 1899).

[Ans. 33 ; 5]

36. A brass ball 7 cm. in radius is suspended concentrically inside a spherical brass vessel of internal radius 9 cm. and external radius 10 cm. If the charge on the ball is 56 units and the potential of the outer vessel 5, what is the potential of the ball ? [B. of E. 1904].

[Ans. 6·7].

37. The difference of potential between two points A and B in a uniform electric field is 100, A and B being 4 cm. apart and lying upon the same line of force. A body charged with 10 units of positive electricity is placed upon the line AB. What force does it experience ? [London University Matric 1906].

[Ans. 250 dynes]

38. Two equal soap-bubbles are equally and similarly charged. Then they coalesce into a single larger bubble. What would be the potential of the bubble formed by their union if the potential of each bubble

while at a distance from the other and from all other conductors was R.

$$\left[\text{Ans. } \frac{2}{(2)^{\frac{1}{3}}} \text{ R.}\right]$$

39. The potentials of two insulated and widely separated metallic spheres are raised to 8 and 10 respectively. The densities of the charges being in the ratio of 2: 3, compare the radii of the balls.

[Ans 1·2 : 1]

CAPACITY

40. A charge of 40 C. G. S. units raises the potential of a spherical conductor through 10 units. Find the radius of the sphere.

[Ans. 4 cm.]

41. A sphere of radius r is surrounded by a concentric sphere of radius r'. If r is earthed and r' insulated, what is the capacity of the condenser so formed ?

$$\left[\text{Ans. } \frac{r'^2}{r'-r}\right]$$

42. At what distance should the plates, 3 cm. in diameter of an air condenser be placed in order to have the same capacity as a sphere 100 cm. in diameter ? [Inter B. Sc.] [Ans. 0·01125.]

43. A sphere of radius 40 millimetres is surrounded by a concentric sphere of radius 42 mm, the space

between the two being filled with air. What is the relation between the capacity of this system and that of another similar system in which the radii of the spheres are 50 and 52 mm. respectively and the space between them is filled with paraffin of specific inductive capacity 2·5? [B. of E 1892]

[Ans. 84 : 325]

44. An air condenser is formed of two circular metal plates each of 5 cm. radius, placed at a distance of 0·5 cm from one another. The collecting plate was charged to a potential 4. What was its charge? [C. of P. Senior 1905]. [Ans. 50].

45. What is the capacity of a condenser made of sheet glass 2 mm. thick with tinfoil coatings each 30 cm. square, if the dielectric constant of glass is 7·5? (L. U. 1904) [Ans. 2685·7 cms].

46. In the above question calculate the capacity of the condenser in microfarad if air is used as dielectric.

(1 micro-farad $= 9 \times 10^5$ e. s. units).

[Ans. 0·000398].

47. A spherical conductor of 4 cm. radius which if isolated, would be at a potential of 200 units, is placed inside an uncharged spherical conductor of 8 cm. radius, and they are brought into contact. Find the potential of the larger sphere.

The smaller sphere is then removed and brought into contact with the larger sphere externally : find the final charge of the smaller sphere.

(Camb. School 1895)

[Ans. (1) 100 (2) 266·6]

48. Calculate the capacity of a parallel plate air condenser of which each plate has an area of 400 square centimetres the distance between the plates being half a millimetre. [B. of E. 1906]

[Ans. 636 $\frac{4}{11}$ e. s. units or cm.].

49. Calculate in terms of microfarad the capacity of an air condenser in which the plates are parallel and each 1 square metre in area, the distance between them being 1 millimetre.

(Vict. B. Sc. 1891). [Ans. 0·267 microfarad].

50. The area of inner coatings of a Leyden jar is 100 sq. cm. The specific inductive capacity of the glass is 4·4 and its thickness 2 mm. What is the radius of the sphere which has the same electric capacity as the Leyden jar ?

[Ans. 17·5 cm.]

51. In a cylindrical Leyden jar the inner diameter of the glass vessel and the height of the inner coating are 10 cm and 15 cm. respectively. Calculate the

capacity of the Leyden jar if the thickness of the glass be 0·2 cm. [Take K = 3].

[Ans. 656·25 C.G.S. units]

52. A Leyden jar A, of capacity 3, is insulated and the outer coating is connected by a wire with the inner coating of another Leyden jar B, of capacity 2, the outer coating of which is uninsulated. If the inner coating of A be charged so that the potential is V, what is the potential of the inner coating of B ?

(B. of E. 1899) [Ans. $\frac{3}{5}$ V]

53. A Leyden jar consists of two concentric spherical surfaces of 5 and 6 centimetres diameter respectively, the intervening space being filled with air. The outer sphere is uninsulated, the inner is charged with 20 units of electricity. How much work is done when the inner sphere is put to the earth ? (B of E. 1895).

[Ans. 13$\frac{1}{3}$ ergs]

54. A Leyden jar has a radius of 7·5 cm; a depth of tinfoil of 18 cm. and thickness of glass 1·25 mm. If the dielectric constant is given to be 3·2, find the capacity of the jar.

[Ans. 2088 C. G. S. units].

55. Calculate the capacity of a cylindrical Leyden jar if the inner radius of the glass vessel is 8 cm., the height of the inner coating 18 cm., and the thickness of the glass 0·2 cm. The value of K is given to be 3. [Ans. 1320 e. s. units]

56. Two parallel plate condensers, are charged respectively with -150 and -75 units of electricity. The area of the plates are respectively 100 sq. cm. and 140 sq. cm. The first one has air as dielectric of 0·5 cm. thickness and the other one has a dielectric of constant 4·2 and thickness 0·6 cm. Which way will the current flow if the two condensers are connected by a wire? Find their common potential.

[Ans. From second to first; $-2{\cdot}397$ e. s. units]

57. A condenser consists of eleven rectangular pieces of tinfoil each measuring 15 cm. × 20 cm. all joined together, with ten similar pieces of tinfoil joined together and alternating with the first set. If the tinfoils are separated by sheets of mica of thickness 0·2 mm. whose specific inductive capacity is 6·28, what is the capacity of the condenser? Also find the amount of work necessary to put a charge of 100 electrostatic units upon it (L. U.)

[Ans. 150000 C. G. S. units; 0·0333 erg]

58. An insulated metal sphere A is positively charged. Another insulated sphere B of equal radius but uncharged is momentarily brought into contact with it and then removed. What will be the ratio of (1) the charge (2) the potential (3) the electric energy of A after contact to the value of each of those quantities before contact?

[Lond. Univ. Matric. 1907]

[Ans. Charge = $\frac{1}{2}$; potential = $\frac{1}{2}$; energy = $\frac{1}{4}$]

59. An air condenser with plates 10 centimetres square and half a centimetre apart is charged with 100 electrostatic units of electricity. Find the loss of electric energy when it is plunged under oil of specific inductive capacity 2. (L. U.) [Ans. 157·1 ergs].

60. Two spheres of radii 20 and 30 cm, have equal charges of 180 units each. They are then joined by a thin wire so that their charges are redistributed over them, Calculate the loss of energy.

[Ans. 54].

61. A spherical condenser has surfaces of 18·5 cm and 18 cm. radius respectively. The insulated sphere receives a charge of 60 units and the space between the spheres is filled by a liquid of dielectric constant 6. Find the change in energy of the charge on running the liquid out from between the spheres.

(U. L. Inter. 1921) [Ans 4·385 ergs gain]

62. The armatures of a condenser are concentric spheres of radius 12 and 15 cm, respectively and the space between them is filled with sulphur of S. I. C. $=2\cdot5$. The condenser has a charge of 2500 e. s. units. How much heat will be produced if the condenser be discharged in such a way that all its energy is converted into heat? [$J = 4\cdot2 \times 10^7$ ergs].

[Ans $\frac{1}{2016}$ calorie].

End

CHAPTER III.

OHM'S LAW.

Ohm in 1827 discovered that the strength of current in any circuit varied directly as the E. M. F. in the circuit and inversely as its total resistance, so long as the physical state of the conductor (temperature etc.) does not change. This is known after him as Ohm's Law.

Expressed in other words we have.

$$\text{Current (C)} = \frac{\text{Electromotive force}}{\text{Resistance}} \quad \textit{i.e.}\ C = \frac{E}{R}.$$

This law is also true of any portion of a circuit and holds good numerically for either practical or absolute units.

Resistance of a conductor is directly proportional to its length and varies inversely as its area of cross section. It also depends upon the nature of the material.

We may express it as

$R = S.\dfrac{L}{A}$ where S is a number which is constant for any given material and L and A are the length and cross-section of the conductor.

If $L=1$ and $A=1$ then $R=S$. *i.e.* S. is the resistance of a portion of the material of unit length and unit

sectional area. It is called the specific resistance of that material, and its numerical value will depend upon the units of length and resistance chosen. If we take *centimetre* as the unit of length and consequently the *square centimetre* as the unit of area then the value of S will be in *ohms per centimetre cube*. In other words the specific resistance is the resistance between the opposite faces of a centimetre cube.

Resistance of certain conductors increases with temperature and in such cases the bodies are said to have positive temperature coefficient. (*e.g* metals and alloys).

Resistance of other bodies (such as carbon, conducting solutions and liquids, boron) decreases with rise of temperature and these bodies are said to have negative temperature coefficient.

If R_o is the resistance of a conductor at $0°C$ snd R_t—the resistance at a temperature $t°C$ then we have $R_t = R_o (1 + \alpha t)$ where α is the temperature coefficient of resistance (which may be positive or negative).

Note:—The practical unit of resistance is called the **ohm.**

It is 10^9 times the *e.m,* unit of resistance and is the resistance of a mercury column 1 square millimetre in section and 106·3 centimetres long at $0°C$. (This is known as **International ohm.**).

Conductors in series : - If conductors of resistances r_1, r_2 and r_3 ohms respectively are joined in series the total resistance (or equivalent resistance) is equal to ths sum of their resistances.

i.e. $R = r_1 + r_2 + r_3$

Conductors in parallel :—If conductors of resistances r_1, r_2 and r_3 ohms be joined in parallel the reciprocal of the equivalent resistance is the sum of the reciprocals of the resistances of separate conductors (in other words equivalent conductance of a number of conductors arranged in parallel is the sum of their individual conductances.

$$\frac{1}{R} = \frac{1}{r_1} + \frac{1}{r_2} + \frac{1}{r_3}.$$

Cells in series and parallel :—

(1) If n cells each of E.M.F. E and internal resistance B be connected in series then their total E.M.F and their total internal resistance are given by nE and nB respectively. If such a grouping of cells sends a current through an external resistance R then we have

$$\text{Current } C = \frac{nE}{R+nB}.$$

Note :—If R be the resistance of a conductor, then $\frac{1}{R}$ is called its conductance.

(2) If *n* such cells be connected in parallel their combined E.M.F. will be the same as that of one cell i e. E, and their total internal resistance is $\frac{1}{n}$ times that of one cell i.e. $\frac{B}{n}$.

Then the current through an external resistance R is given by $C = \dfrac{E}{R + \frac{B}{n}}$

Mixed grouping of cells :—

If there are m rows of cells connected in parallel and each row consisting of n cells in series, the current through an external resistance R is given by

$$C = \frac{nE}{\frac{nB}{m} + R}$$

Maximum current is obtained when $R = \frac{nB}{m}$ i. e. when the cells are arranged in such a manner that the internal resistance of the battery is equal to the external resistance.

Kirchoff's Laws :—

(1) In any branching net-work of wires, the algebraic sum of the currents in all the wires meeting at a point is zero.

(2) In any closed path in the net-work, the algebraic sum of the C × R products is equal to the E.M.F. acting in that path.

In applying the above laws the following rules should be observed

If a current be flowing *towards* a point it is taken as *positive* and if flowing away it is taken as *negative*. The *clock-wise* cyclic currents in any closed path in the net work are reckoned as *positive* and *anti-clockwise* currents *negative*

The End The End

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Solved Examples.

1. A current of 0·6 ampere is sent through a 100 volt glow-lamp. Find the resistance of the lamp.

Since C is expressed in amperes and E in volts, the resistance of the lamp in ohms is given by the equation $R = E/C = 100/{\cdot}6 = 166\frac{2}{3}$ ohms.

2. The E.M.F. between the poles of a battery is 12 volts when it is an open circuit. When the poles are connected by a resistance wire the potential difference between the battery poles falls to 10 volts. Find the resistance of the wire and the internal resistance of the battery if the current running through the wire is 5 amperes.

The Resistance of the wire is the ratio of the difference of potential between its ends (10 volts) to the current running through it (5 amperes)

$$\therefore R = \frac{E}{C} = \frac{10}{5} = 2 \text{ ohms.}$$

The total E. M. F. acting round the circuit is 12 volts. It falls to 10 volts causing a flow of current (5 amp) through the wire of resistance 2 ohms.

Applying ohm's law to the complete circuit we have $E = CR = C(R_1 + B)$

or $12 = \cdot 5(2+B)$ where B is the internal resistance of the battery.

$\therefore B = \frac{2}{5} = \cdot 4$ ohm.

3. A Daniell cell, the internal resistance of which is 0·3 ohm, works through an external resistance of 1 ohm. What must be the resistance of another Daniell cell, so that when it is joined up in series with the first and working through the same external resistance the current shall be the same as before. If the cells be joined up in parallel, how will the current be modified?

(S. K. 1893)

We know that $C = \frac{E}{R+B_1}$

In the first case we shall have $C = \frac{E}{1+\cdot 3}$ ... (i).

When the second Daniell cell is joined up in series we shall have $C = \frac{2E}{1+\cdot 3+B_2}$ .. (ii).

From the equations (i) and (ii) we get

$$\frac{E}{1\cdot 3} = \frac{2E}{1\cdot 3+B_2}$$

$\therefore B_2 = 1\cdot 3$ ohms.

(b) When the cells are joined in parallel the E. M. F. of the cells will be the same as that of one

cell but the internal resistance B of the battery is given by $\frac{1}{B}=\frac{1}{B_1}+\frac{1}{B_2}$

$$=\frac{1}{\cdot 3}+\frac{1}{1\cdot 3}$$

$$\therefore \quad B=\frac{\cdot 39}{1\cdot 6}=\frac{39}{160} \text{ ohm.}$$

If C_1 is the new current in this case then we shall have

$$C_1=E/(1+B)=E/\left(1+\frac{39}{160}\right)=\frac{160\ E}{199} \quad .. \quad \text{(iii)}$$

From equations (ii) and (iii) we get

$$\frac{C}{C_1}=\frac{2E}{1\cdot 3+1\cdot 3}\div\frac{160\ E}{199}=\frac{2E}{2\cdot 6}\times\frac{199}{160E}$$

$$=\frac{199}{208}.$$

Thus the ratio of the currents in the two cases will be $C: C_1=199: 208$.

4. Calculate the number of cells required to produce a current of 50 milli-amperes through a line 114 miles long, whose resistance is 12·5 ohms per mile, the available cells of the battery having each an internal resistance of 1·5 ohms and an E. M. F. of 1·5 volts. (Patna University).

Suppose n is the number of cells required. The E.M.F. of the battery is $1 \cdot 5\, n$ volts and its internal resistance is $1 \cdot 5\, n$ ohms. The resistance of the line is $114 \times 12 \cdot 5 = 1425$ ohms.

The total external resistance = 1425 ohms (the resistance of the return circuit through the earth is negligible).

A milli-ampere is one-thousandth of an ampere and hence 50 milli-amperes = $50/1000 = \cdot 05$ amp.

By ohm's law we know tnat

$$C = E/(R+B)$$

i.e. $$\cdot 05 = 1 \cdot 5\, n/(1425 + 1 \cdot 5\, n)$$

$$\text{or } \cdot 05\,(1425 + 1 \cdot 5\, n) = 1 \cdot 5\, n.$$

$$\text{or } 1 \cdot 5 n - \cdot 075 n = 71 \cdot 25.$$

$$\text{or } 1 \cdot 425\, n = 71 \cdot 25.$$

$$\therefore n = 71 \cdot 25/1 \cdot 425 = 71250/1425 = 50.$$

Thus the number of cells required will be 50.

5. A copper wire 300 cm. long and 0·02 cm. in radius is found to have a resistance of 0·392 ohm. Calculate the specific resistance of copper, (a) in ohms per centimetre-cube, (b) in absolute units of resistance per centimetre-cube, (c) in microhms per inch-cube.

We know that

$$R = S \times \frac{L}{A}.$$

$$\therefore S = \frac{R \times A}{L}$$

$\therefore$ In case (a)

$$S = \frac{0{\cdot}392 \times \pi \times (0{\cdot}02)^2}{300}$$

($\because$ area of the cross-section *i.e.* $A = \pi r^2$)

or $S = {\cdot}00000\,1643$,

$\doteq 1{\cdot}643 \times 10^{-6}$ ohms per centimetre-cube.

For the second case we have to take R in absolute units *i.e.* we shall have to multiply it by 10^9.

Then $S = 1{\cdot}643 \times 10^{-6} \times 10^9$

$= 1643$ absolute units per centimetre-cube.

For the third case we have first to express the dimensions in inches.

We know that

$2{\cdot}54$ cm. $= 1$ inch.

$\therefore$ 300 cm. $= \frac{300}{2{\cdot}54}$ inches

and $\cdot 02$ cm. $= \frac{\cdot 02}{2\cdot 54}$ inch

$$\therefore \quad S = \frac{\cdot 392 \times \pi \left(\frac{\cdot 02}{2\cdot 54}\right)^2}{\frac{300}{2\cdot 54}}$$

$$= 00000065$$

$$= \cdot 65 \times 10^{-6} \text{ ohm per inch-cube.}$$

$$= 0\cdot 65 \text{ microhm per inch-cube.}$$

$$\left(\text{Since } 1 \text{ microhm} = \frac{1}{10^6} \text{ ohm}\right)$$

6. The same current passes through a metre of copper wire 1 mm. diameter and two metres of thinner copper wire. The difference of potential between the ends of the first wire is 1 volt and that between the ends of the second wire 20 volts. Find the diameter of the thinner wire. [Inter. Board. 1922].

We know that $R = \frac{E}{C}$ (Ohm's law.) Resistances of the two wires are therefore given by.

$$R_1 = \frac{1}{C}$$

and $R_2 = \frac{20}{C}$

Hence $\frac{R_1}{R_2} = \frac{1}{20}$.

$$\text{But } \frac{R_1}{R_2} = \frac{S.\dfrac{L_1}{A_1}}{S\dfrac{L_2}{A_2}} = \frac{L_1\, A_2}{L_2\, A_1}$$

$$\therefore \frac{L_1\, A_2}{L_2\, A_1} = \frac{1}{20}$$

$$\text{or } \frac{1\ \pi r^2}{2 \times \pi (\cdot 05)^2} = \frac{1}{20}$$

or $r = 0{\cdot}016$ cm (approx).

$\therefore$ The diameter of the given wire $= 0{\cdot}032$ cm.

7. A cell tested by a 30 ohm voltmeter gives a reading of $1{\cdot}90$ volts. On shunting the voltmeter with a 10 ohm coil the reading falls to $1{\cdot}65$ volts. What is the e.m.f. of the cell? (U. L. Inter. 1926).

Let E be the electromotive force and B the internal resistance of the battery.

Then we have in the first case,

$$\frac{E}{B+30} = C_1 \ldots\ldots\ldots (1)$$

and in the second case,

$$\frac{E}{\dfrac{1}{\left(\dfrac{1}{30}+\dfrac{1}{10}\right)} + B} = C_2 \ldots\ldots\ldots\ldots (2)$$

(Since the equivalent resistance when the voltmeter is shunted is given by

$$\frac{1}{R} = \frac{1}{30} + \frac{1}{10} = \frac{2}{15})$$

Now we know that,

$$C_1 = \frac{E_1}{R_1} = \frac{1 \cdot 9}{30}, \text{ and } C_2 = \frac{E_2}{R_2} = \frac{1 \cdot 65}{\frac{15}{2}}$$

Substituting the values of C_1 and C_2 in the equations (1) and (2) we get,

$$\frac{E}{B+30} = \frac{1 \cdot 9}{30} \quad \ldots\ldots\ldots (3)$$

$$\text{and} \quad \frac{E}{\frac{15}{2}+B} = \frac{1 \cdot 65}{15/2} \quad \ldots\ldots\ldots (4)$$

Solving these equations we get $E = 1 \cdot 65 + \cdot 351$
$= 2 \cdot 001$ volts.

8. A battery, of negligible resistance and an E. M. F. of 4 volts, is connected to the opposite corners A and C of a quadrilateral wire frame ABCD, The resistance of the side AB = 90 ohms, of BC = 110 ohms, of CD = 60 ohms, and of DA = 40 ohms. Calculate the P. D. between the points B and D. (U. L)

In the circuit A B C the combined resistance $= 90 + 110 = 200$ ohms.

But E is given to be 4 volts.

∴ The current through the circuit A B C is given by $C_1 = \frac{E}{R_1} = \frac{4}{200} = \frac{1}{50}$ amp.

Similarly the current through the circuit ADC is given by

$$C_2 = \frac{E}{R_2} = \frac{4}{100} = \frac{1}{25} \text{ amp.}$$

In order to find the fall of potential between the points A and B we have to multiply the current in that circuit by the resistance of the arm AB.

Hence the difference of potential between the points A and B $= \frac{90 \times 1}{50} = 1{\cdot}8$ volts. Similarly the difference of potential between the points A and D

$$= \frac{40 \times 2}{50} = 1{\cdot}6 \text{ volts.}$$

Hence the difference of potential between the the points B and D.

$$= 1{\cdot}8 - 1{\cdot}6$$

$$= {\cdot}2 \text{ volt.}$$

9. How would you arrange 30 cells in each of which the resistance is 5 ohms so as to send the more powerful current through an external circuit of 6 ohms resistance. (I. B. U. P. 1917)

In order to send maximum current through the circuit we must have

$R = \frac{mB}{n}$ where R is the external resistance, B the internal resistance, n—the number of rows arranged in parallel and m—the number cells in series in each row.

Hence in the question we must have

$$6 = \frac{m}{n} \times 5$$

$$\text{or} \frac{m}{n} = \frac{6}{5} \ldots\ldots\ldots\ldots\ldots\ldots (1)$$

$$\text{But } m \times n = 30 \ldots\ldots\ldots\ldots\ldots\ldots (2)$$

From the equations (1) and (2)

we get $m = 6$

and $n = 5$.

10. Two batteries of E. M. F.'s 1·1 and 1·4 and resistances 4 and 6 ohms respectively are arrang-

ed in parallel so that they tend to send currents in the same direction through an external resistance of 5 ohms. Calculate the strength of the current flowing through the external resistance.

Let us first take the most general case.

Let C be the current through the external resistance, c_1 and c_2 be the currents through the two batteries and b_1 and b_2 their internal resistances.

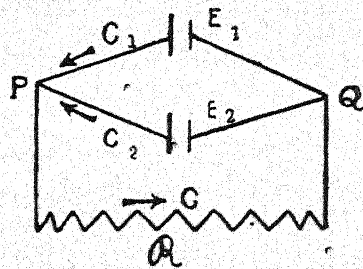


Fig 5.

Applying Kirchhoff's second law, we have in the circuit E_1 PRQ

$$c_1 b_1 + CR = E_1 \ldots (1)$$

and in the circuit E_2PRQ,

$$c_2 b_2 + CR = E_2 \ldots\ldots (2)$$

By Kirchhoff's first law we also know that

$$C = c_1 + c_2$$

$\therefore$ From equations (1) and (2) we get

$$C = \frac{E_1 - CR}{b_1} + \frac{E_2 - CR}{b_2}$$

Simplifying we get

$$C = \frac{E_1 b_2 + E_2 b_1}{b_1 b_2 + b_1 R + b_2 R} \quad \ldots\ldots\ldots\ldots (3)$$

Substituting the given values in the equation (3) we get

$$C = \frac{1{\cdot}1 \times 6 + 1{\cdot}4 \times 4}{4 \times 6 + 4 \times 5 + 6 \times 5} = \frac{6{\cdot}6 + 5{\cdot}6}{24 + 20 + 30}$$

$$= \frac{12{\cdot}2}{74} = \frac{6{\cdot}1}{37} \text{ amp.}$$

The End

Exercises.

8 1. What E. M. F. is required to work an incandescent lamp of 110 ohms resistance which takes a current of 2 amperes ? [Ans. 220 volts]

8 2. An incandescent lamp is connected to mains which gives 440 volts. If the lamp takes a current of 8 amperes, find its resistance when hot.

[Ans. 55 ohms]

×8 3. A battery consists of 6 Leclanche cells, each having an E. M. F of 1·4 volts and an internal resistance of 6 ohms; what current will the battery produce with an external resistance of 12·0 ohms ?

(Ans. $\frac{7}{40}$ amp.)

4. A cell of electromotive force of 2 volts and internal resistance $\frac{1}{2}$ ohm is used to send a current through a wire of $11\frac{1}{2}$ ohms resistance. Find the strength of the current.

[Oxford Local Senior 1899]. [Ans. $\frac{1}{6}$ ampere].

5. The E. M. F. of a battery being 15 volts and its resistance 10 ohms, find the strength of the current generated by it when its poles are connected by a resistance of 20 ohms.

[Ans. 0·5 amp]

6. If an increase of the resistance of a circuit by 12 ohms causes the strength of the current to decrease from 5 to 3 amperes, find the total resistance of the circuit after the change.

[Ans. 30 ohms]

7. A battery of ten cells in series yields a current of 1 ampere, when the external resistance is 10 ohms, and a current of 0·6 amp, when the external resistance is 20 ohms. Find the E. M. F. and the internal resistance of one of the cells (these being the same for all). (Cal. University).

[Ans. 1·5 volts; 0·5 ohm].

8. When the poles of a voltaic cell are connected by a coductor of resistance 1, a current of strength 1·32 is produced; and when they are connected by a conductor of resistance 5 the strength of the current is 0·33. Find from these data the internal resistance and the electromotive force of the cell. (Patna University).

[Ans. $R = \frac{1}{3}$ ohm; $E = 1·76$ volts]

9. If we have 5 cells each with an E. M F. of 2 volts and $\frac{1}{4}$ ohm internal resistance, what current, in amperes, shall we get if we place the cells in series

and connect the poles with a wire whose resistance is $\frac{1}{2}$ ohm ? (Oxf. Loc. 1908).

[Ans. $5\frac{5}{7}$ amperes]

10. A cell, whose resistance is ·02 ohm and whose E.M.F. is 2 volts has its poles connected by a wire; what must be the resistance of this wire in order that the current produced may be 10 amperes? (Oxf. Loc. 1904).

[Ans. 0·18 ohm]

11. The E.M.F. between the poles of a battery is 12 volts when the circuit is "open" and 10 volts when it is closed by a resistance such that a current of 6 amperes is passing. Find the resistance of the battery. (Camb. L. S. 1895)

[Ans. $\frac{1}{3}$ ohm]

12. The terminals of a battery of 6 Grove cells, the total E M.F. of which is 10·8 volts are connected by three wires, the resistance of each of which is 9 ohms. The current through each wire is $\frac{6}{7}$ of an ampere. What is the internal resistance of each cell?

[Ans. 0·2 ohm]

13. Two electric batteries have electromotive forces of 14 volts and 11 volts and resistances of 8 ohms and 2 ohms respectively. It is found that they giev the same current through a certain wire. Find

give

the resistance of this wire and the strength of the current in amperes. (Oxf. Prelim. 1909).

[Ans. 20 ohm ; 0·5 ampere].

14. A battery of six similar cells connected in series sends a current of 2 amperes through a coil of wire the resistance of which is 5 ohms. The E.M.F. of the battery is 12·6 volts. What is the internal resistance of each cell ? [Camb-Loc. 1907].

$\left[\text{Ans. } \frac{13}{60} \text{ ohm.}\right]$

15. The poles of a given constant battery are joined by a wire of 1 ohm resistance, and the potential difference between them is then 1 volt. A second wire of 3 ohms resistance is now joined in parallel with the first, and the potential difference between the poles of the battery falls to 0·9 volt. Find the electromotive force and the internal resistance of the battery. (L U.)

[Ans. 0·5 ohm ; 1·5 volts.].

16. The current in a certain circuit is diminished in the ratio of 10: 9 by adding a resistance of 5 ohms Find out the original resistance. How much should be added to this in order to bring the current down to half its original value ?

[Ans. (1) 45 ohms; (2) 45 ohms more]

17. How many cells, each of 1·4 volts E.M.F. and 5 ohms internal resistance will be required to send a current of 0·08 ampere through an external resistance of 100 ohms ?

[Ans. 8 cells].

18. The potential difference between the poles of a battery (of 1·2 ohms resistance) is 6 volts when the poles are insulated, and 4·5 volts when they are joined by a wire. What is the resistance of the wire ? (L. U)

(Ans. 3·6 ohms)

19. A circuit is formed of six similar cells in series and a wire of 10 ohms resistance. The E.M F. of each cell is 1 volt and its internal resistance 5 ohms. Determine the difference of potential between the positive and negative poles of any one of the cells. (B of E. 1894)

[Ans. $\frac{1}{4}$ volt.]

20. Six cells arranged in series, each having an internal resistance of 0·4 ohm, are connected by a wire of 1·6 ohms. If each cell has an electromotive force of 1 volt what is the potential difference between the positive pole of the battery and the point of junction of the third and fourth cells ? (B. of E. 1899).

[Ans 1·2 volts]

21. Show that if the external resistance is equal to the resistance of a cell, the same current strength is produced whether the battery is coupled in series or in parallel. (Camb Local 1907)

22. A battery of 10 volts and internal resistance ·5 ohm is connected in parallel with one of 12 volts and internal resistance 0·8 ohm. The poles are connected to an external resistance of 20 ohms. Find the current in each branch. (London University B. Sc. 1907).

[Ans. $C_1 = 115/66$ ampere, $C_2 = 80/66$ ampere flowing backwards through weaker battery].

23. Four points A, B, C and D are connected together as follows: A to B, B to C, C to D, D to A, each by a wire of 1 ohm resistance; A to C, B to D, each by a cell of 1 volt E.M.F and 2 ohms resistance. Determine the current flowing through each of the cells. (B. of E. 1900)

[Ans. $\frac{1}{3}$ ampere]

24. A battery of 20 volts E.M.F. and 4 ohms resistance is joined in parallel with another of 20 volts and 3 ohms to send a current through an external resistance of 10 ohms. Calculate the current through each battery. (L. U.)

[Ans. 0·976; 0·732 amp]

25. Six Daniell cells for each of which $E = 1\cdot05$ volts, $r = 0\cdot5$ ohm are joined in series. Three wires X, Y and Z whose resistances are severally 3, 30 and 300 ohms, can be inserted between the poles of the battery. Determine the current which flows when each wire is inserted separately; also determine that which flows when they are all inserted at once in parallel. (Patna University).

[Ans. Through X, 1·05 amperes. Through Y, 0·1909 ampere. Through Z 0·0207 ampere. Through all three 1·105 amp].

26. Calculate the combined resistance of four conductors in parallel of resistances 4, 8, 12, and 24 ohms respectively. If the total current is 2·4 amperes, find the current in the 12 ohm conductor.

[Ans. 2 ohms; 0·4 ampere].

27. The E. M. F's of two cells are 2 volts and 1 volt and their internal resistances 1 ohm and 2 ohms respectively. The cells are connected in parallel and their terminals joined by a resistance of 4 ohms. Find the current in each branch of the circuit (U. L. Inter. Eng, 1928)

[Ans. 0·571, −0·214, 0·357 amp]

28. Four cells each of 1·5 volts e. m. f. and 2 ohms internal resistance, are used to send a

current through a single wire of 2 ohms resistance. The cells are arranged (a) all in series, (b) in two parallel groups of two in series, and (c) all in parallel. Calculate the current in the wire in each case. (L. U.)

[Ans. (a) 0·6 amp. (b) 0·75 amp. (c) 0·6 amp.]

29. Two cells, each of E.M.F. 1·5 volts and resistance 5 ohms, are joined in series with a resistance box and a resistance coil of 10 ohms. What resistance must be unplugged in the box inorder that the difference of potential between the ends of the 10 ohm coil may be 0·1 volt? (Madras University)

[Ans. 280 ohms.]

30. A uniform wire, 4 metres long, whose resistance per metre is 6 ohms, is bent into the form of a square ABCD. The adjacent points A and B are fixed to the poles of a battery of E.M.F. 3 volts and internal resistance 4 ohms. Find the current along AB and CD; also the current between the plates of the battery. (Camb. School 1895)

[Ans. $\frac{6}{17}$ ampere through the battery, $\frac{9}{34}$ amp. through AB; and $\frac{3}{34}$ ampere through CD.]

31. Given a voltmeter of resistance x ohms, reading from $0-1\cdot5$ volts, how would you use it to measure voltages up to 150 volts ? (U. L.)

[Ans. 99 x ohms in series]

32. Four resistances equal to 10 ohms, 100 ohms, 60 ohms and 600 ohms, form the arms of a Wheatstone bridge. If the voltage of cell connected to the bridge be $1\cdot5$ volts, find the current in each arm, (Inter. Board. U.P. 1932]

$$\left| \text{Ans. } \frac{3}{220} \text{ amp; } \frac{1}{440} \text{ amp.} \right|$$

33. In a Wheatstone's bridge in which resistances of 100 and 10 ohms respectively were used as the fixed resistances, a wire whose resistance was to be determined was placed : its resistance was balanced when the adjustable coils were arranged to throw 281 ohms into circuit. What was its resistance ? (Patna University).

[Ans. $28\cdot1$ ohms]

34. A battery of fifty-four cells each of E. M. F. 2 volts and resistance $0\cdot005$ ohm is employed with a number of 100-volt glow lamps all in parallel, each lamp requiring $\cdot6$ ampere ; what is the maximum number of lamps which can be used so that the voltage at their terminals shall not fall below 100 volts ? (Oxford Local 1907).

[Ans. 49 lamps]

35. An electric light installation consists of a group of lamps in parallel arc between the ends of leads. The leads have total resistance of 0·4 ohm and bring current from 60 accumulators each with E. M. F. 2 volts and resistance ·01 ohm. When twenty-five lamps are switched on, each takes 0·4 amp. Find the resistance of a lamp and the watts used in each part of the circuit. (Lond. University 1894).

[Ans. 275 ohms ; 60 watts in battery, 40 watts in leads, 1100 watts in lamps].

36. The positive poles W and Y of two cells are connected by a resistance of 10 ohms and their negative poles X and Z by a resistance of 20 ohms. If the middle point of W Y is connected to earth, what will be the potential of the middle point of XZ.?

The E. M. F. of W X is 2 volts, its resistance 2 ohms. The E.M.F. of YZ is 1 volt, its resistance 1 ohm. (U. L. Inter. 1927).

[Ans. — 1·485 volts].

37. The terminals of a battery, of negligible resistance, are connected by a wire of 100 ohms resistance which is 2 metres long. A voltmeter of 50 ohms resistance, placed as a shunt across 1 metre of

the wire, indicates 0·7 volt. What is the electromotive force of the battery? (U. L. Inter. 1925).

[Ans. 2·1 volts.]

38. How would you connect two equal constant cells of internal resistance 5 ohms each, if you wished to deposit copper as rapidly as possible in a voltameter of 7 ohms resistance? (B of E. 1906).

[Ans. In series].

39. Four cells each of 1 ohm internal resistance are coupled up in series with an external resistance of 8 ohms. Two similar cells are coupled up in parallel with an external resistance of 5·5 ohms. Compare the currents in the two circuits. (Oxf. Local. 1901.)

[Ans. 2 : 1].

40. A wire 30 centimetres long and ·02 centimetre in radius is found to have a resistance of 0·0392 ohm. Find the specific resistance of the wire.

[Ans. $1·64 \times 10^{-6}$ ohms per centimetre-cube]

41. A mile of telegraph wire 1 mm. in radius offers a resistance of 26 ohms; what is the resistance of 440 yds. of wire 0·4 mm. in radius made of the same material?

[Ans. 40·62 ohms]

42. What length of copper wire, having a radius of 1·5 millimetres has the same resistance as 20 metres of copper wire, having a radius equal to 1 millimetre ?

[Ans 45 metres.]

43. If the resistance of two yards of iron wire, 0·015 inch in radius be 0·394 ohm, what is the resistance of 15 miles of iron wire, 0·3 inch in diameter ?

[Ans. 52·008 ohms.]

44. Find the length of wire of diameter 0·3 mm. and resistivity $1·8 \times 10^{-6}$ ohm-centimetre which would have a resistance of 2 ohms. (U. L)

[Ans. 785·2 cm]

45. A coil concealed in a box but with its terminals exposed, is found to have a resistance of 126 ohms. A piece of wire, 10 metres in length, of the same material as that of the coil, but of twice its cross-sectional area, is found to have a resistance of 12 ohms. What is the length of the wire in the coil ? Lond. Univ. Matric. 1900.)

] Ans. 52·5 metres.]

46. A piece of wire 50 yards long weighs 0·5 lb ; another piece of the same wire 250 yards long weighs 0·125 lb. Compare their resistances.

[Ans. 1 : 100].

47. One kilogram of copper is drawn into wire (*a*) 1 mm. thick (*b*) 2 mms. thick. Compare their respective resistances. (Inter. Board. U.P. 1926.)

[Ans. 16 : 1].

48. The bronge wire used by Post Office for overhead telephone circuits has specific resistance 3·37 micro-ohms. The diameter of the wire is 0·122 cm. Find the resistance in ohms of a mile of the wire. (Inter. Board. 1928.)

Ans. 46·4 ohms.]

49. 660 yards of iron wire 0·0625 of an inch in diameter have the same electrical resistance as a mile of copper wire 0·0416 of an inch in diameter. Find the specific resistance of iron, that of copper being unity. (Allahabad University.)

[Ans. 6·02.]

50. On the bobbins of a single needle telegraph are coiled 225 yards of No. 35 copper wire 0·0087 inch in diameter, the resistance of which is about 92 ohms. Required the conducting power of the wire in terms of mercury. (Patna University.)

[Ans. 55·6]

51. If the resistance of 130 yards of a particular copper wire $\frac{1}{16}$ of an inch in diameter is an ohm,

express in that unit the resistance of 8242 yards of copper wire $\frac{1}{12}$ of an inch in diameter. (Patna University.)

[Ans. 35·66.]

52. A battery of 24 Daniell cells each of 3 ohms internal resistance, is required to send the maximum current through an external resistance of 30 ohms: how are the cells to be arranged ? Find also the current produced and the difference of potential between the poles of the battery, assuming that the E. M. F. of a Daniell cell is 1·1 volts.

(Ans. (1) 2 rows of 12 cells each.
(2) 0·22 ampere.
(3) 6·6 volts.)

53. How would you arrange 36 cells, each having a resistance 1·4 ohms, so as to send the strongest possible current through an external resistance of 5·6 ohms ? [U. L.]

[Ans. 3 rows in parallel, each row of 12 in series.]

The End

CHAPTER IV.

Magnetic effects of current.

In the neighbourhood of a conductor carrying a current there is a magnetic field created.

(1) The intensity of the field at any point due to to a current in a straight wire is given by,

$F = \frac{A. S. \sin\theta}{R^2}$ (where A is the strength of the current in e. m. units, S—the length of the element, R— the distance of the element from the point in question and θ, the angle between R and S.

If the wire be infinitely long then we have

$$F = \frac{2A}{R}$$

If A is measured in amperes then,

$$F = \frac{2A}{10R}.$$

(2) Field at the centre of a circular coil carrying current is given by,

$F = \frac{2\pi A}{R}$ (where A is the current in e. m. units, R — the radius of the coil.)

[See Willow's Book p 364].

In the above equation we see that if A and R are each unity, $F = 2\pi$. Thus an electromagnetic unit of current is that current whtch flowing through a circle

of 1 cm. radius produces a magnetic field of 2π dynes at its centre.

(3) Field at any point on the axis of a circular coil is given by,

$$F = \frac{2\pi R^2 A}{(x^2+R^2)^{\frac{3}{2}}}$$ (See Willow's Book p. 368).

(4) Field in the interior of a long closely wound coil solenoid is given by,

$$F = \frac{4\pi N.A}{l}$$ (where A is the current in e. m. units, N the total number of turns of the solenoid, and l its length.

The field is uniform except near the ends where it is half the above value.

If n represents the number of turns per unit length of the solenoid, then $F = 4\pi n A$.

(5) In the case of a Tangent Galvanometer we have,

(i) $A = \frac{R\,H}{2\pi n} \tan\theta$, (where A is the current in e. m. units and θ — the deflection of the galvanometer).

(ii) $A = \frac{10RH}{2\pi n} \tan\theta$ (where A is the current in amp)

$$= K \tan\theta \text{ (where } K = \frac{10RH}{2\pi n}\text{)}$$

This K is called the reduction factor of the galvanometer. In the above formula if $\theta = 45°$ then $K = A$ (since $\tan 45° = 1$).

Thus K—the reduction factor of the galvanometer can also be defined as that current which would produce a deflection of 45° in the galvanometer. (See Willow's Book p. 366)

(6) Current passing through a galvanometer when it is shunted.

If $C =$ total current, $C_g =$ current through the galvanometer, $C_s =$ current through the shunt, and s g be the resistances of the shunt and galvanometer respectively, then we have

$$C_g = \frac{s}{s+g} . C$$

$$C_s = \frac{g}{s+g} . C.$$

(7) Resistance of the Coils of a Tangent Galvanometer is given by

$$G = \frac{R_1 \tan \theta_1 - R_2 \tan \theta_2}{\tan \theta_2 - \tan \theta_1}$$

where R_1 and R_2 are resistances and θ_1 and θ_2, the respective deflections.

(8) The ratio of the E.M.F.'s of two cells

using a Tangent Galvanometer is given by

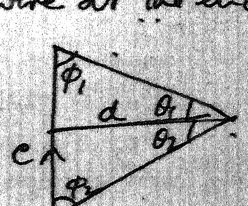
$$\frac{E_1}{E_2} = \frac{\tan\theta_1 + \tan\theta_2}{\tan\theta_1 - \tan\theta_2}$$ where θ_1 and θ_2 are the deflections in the two cases.

(9) The internal resistance of a cell by means of a Potentiometer is given by

$$B = R\left[\frac{l_1 - l_2}{l_2}\right]$$ where l_1 is the first length of the wire, R is the resistance across the poles of the cell and l_2 the corresponding length.

N. B. The intensity of the field at any point d cm. due to a current in a straight wire is given by

$$F = \frac{c}{d}\left[\cos\phi_1 + \cos\phi_2\right]$$ where c is the current in e. m. units and ϕ_1, ϕ_2 are the angles that the radius vector makes with the wire at the ends (See Fig)



or $$F = \frac{c'}{10d}\left[\sin\theta_1 + \sin\theta_2\right]$$

where c' is in amperes and θ_1 and θ_2 are angles complementary to ϕ_1 and ϕ_2.

Solved Examples.

1. A wire of 60 cm. length is placed in a vertical position. A current of 5 amperes runs upwards through it. Calculate the strength of the magnetic field at a distance of 40 cm. from the middle of the wire.

We know that the magnetic field is given by

$F = \frac{A.S \sin\theta}{R^2}$ where ~~S sin θ is the apparent~~ length of the wire.

Use the formula $F = \frac{c}{10d}\left[\sin\theta_1 + \sin\theta_2\right]$ See Fig (a).

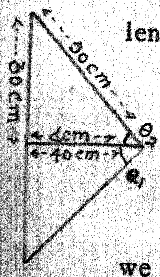


Fig (a)

5 amperes $= \frac{5}{10}$ e. m. unit.

Substituting the values of the known quantities we get $F = \frac{5}{10\times 40}\cdot\left[\frac{3}{5}+\frac{3}{5}\right]$ $= \frac{3}{200} = \cdot015$ Gauss

θ_1 sin θ is found to be $\frac{40}{50} = \frac{4}{5}$.

Substituting these values in ~~the above equation~~ we get.

$$F = \frac{5}{10} \times 50 \times \frac{4}{5} \times \frac{1}{40\times 40}$$

$$= \frac{1}{80} \text{ Gauss.}$$

$$= \cdot0125 \text{ Gauss.}$$

2. Two long straight parallel wires, 10 cm. apart, carry currents of 5 absolute units. Calculate the strength of magnetic field at a point midway between the wires when currents flow in them in opposite (2) in the same, direction.

We know that in the case of a long vertical wire

$$F = \frac{2\,A}{R}$$

$$= \frac{2 \times 5}{5} \doteq 2 \text{ Gauss.}$$

In the first case the currents are flowing in the opposite direction and so the field will be additive.

Since the point in question is equidistant from the two wires, therefore the resultant field will be twice the field due to one wire *i.e.* $F = 2 \times 2 = 4$ Gauss.

In the second, case since the currents are flowing in the same direction the field due to either wire will be in the opposite direction. The two fields being equal at the point mid-way between the wires the resultant field will be zero in this case.

3. A small magnet is placed 10 cm. due magnetic east of a long vertical wire and is found to make 24 oscillations per minute under the inflnence of the earth's horizontal field (0·2 gauss). How many oscillations per minute will it make when a current of 5 amperes runs (1) upwards, (2) downwards, in the wire?

Let F be the field produced by the wire carrying a current of 5 amperes.

Since the wire is long, F is given by

$$F = \frac{2\,A}{R}$$

$$= \frac{2 \times 5/10}{10} \quad [\because 5 \text{ amp} = \frac{5}{10} \text{ e.m. unit.}]$$

$$= \frac{1}{10} = \cdot 1 \text{ gauss.}$$

(1) When the current is running upwards the field will be F+H.

We know that

$$t_1 = 2\pi\sqrt{\frac{K}{MH}}$$

$$\text{or } t_1^2 = 4\pi^2 \frac{K}{MH} \quad \dots \quad \dots \quad (1)$$

$$\text{and } t_2 = 2\pi\sqrt{\frac{K}{M(H+F)}}$$

$$\text{or } t_2^2 = 4\pi^2 \frac{K}{M(H+F)} \quad \dots \quad \dots \quad (2)$$

From (1) and (2) we get

$$\frac{H+F}{H} = \frac{t_1^2}{t_2^2} = \frac{n_2^2}{n_1^2} \left(\because \frac{1}{t} = n\right)$$

Substituting the given values we get

$$\frac{\cdot 2 + \cdot 1}{\cdot 2} = \frac{n_2^2}{24^2}$$

whence $n_2 = 29 \cdot 4$ oscillations per min.

(2) In this case the field = H − F.

Hence as before

$$\frac{H-F}{H} = \frac{n_3^2}{n_1^2}$$

$$\text{or}\quad \frac{\cdot 2 - \cdot 1}{\cdot 2} = \frac{n_3^2}{24^2}$$

whence $n_3 = 16{\cdot}97$ oscillations per min.

4. Calculate the strength of the current in C.G.S. units, and also in amperes from the following data: Radius of coil 12 cm; number of turns in coil 10; deflection of needle 45°; value of earth's horizontal force 0·18. (B. of E 1902).

In C.G.S. units we know that

$$A = \frac{rH}{2\pi n} \tan\theta.$$

Substituting the given values we get

$$A = \frac{12 \times 0{\cdot}18}{2 \times \pi \times 10} \tan 45^\circ$$

$$= \frac{\cdot 378}{11} \times 1 \text{ (for } \tan 45^\circ = 1)$$

$$= \cdot 034. \text{ C.G S. unit.}$$

Since one C.G S. unit = 10 amperes,

∴ the current in amperes will be ·034 × 10 = ·34

5. Three wires of resistances 2, 6 and 12 ohms respectively are connected in parallel and are inserted in a circuit with a cell and tangent galvanometer. The deflection is 60°. The two ohm wire is removed and the deflection becomes 45.° Calculate the resistance of the galvanometer. (Neglect the resistance of the cell). [I. B. 1925.]

We know that the combined resistance in parallel is given by

$$\frac{1}{R_1}=\frac{1}{r_1}+\frac{1}{r_2}+\frac{1}{r_3}$$

$$=\frac{1}{2}+\frac{1}{6}+\frac{1}{12}$$

$$=\tfrac{3}{4}$$

$$\therefore\ R=\tfrac{4}{3}\text{ ohms.}$$

In the second case the combined resistance is given by

$$\frac{1}{R_2}=\frac{1}{r_1}+\frac{1}{r_2}=\frac{1}{6}+\frac{1}{12}=\frac{1}{4}$$

$$\therefore\ R_2=4\text{ ohms.}$$

If G be the resistance of the galvanometer we get the following two equations :—

$$(1)\quad C_1=\frac{E}{R_1+G}=K\tan\theta_1$$

$$\text{or}\quad \frac{E}{\tfrac{4}{3}+G}=K\tan 60^\circ$$

$$\text{or}\quad \frac{E}{\tfrac{1}{3}(4+3G)}=K.\sqrt{3}\ \ldots\ldots\ldots\ldots\ldots\ldots\ (i)$$

$$(\because\ \tan 60^\circ=\sqrt{3})$$

$$(2)\quad C_2=\frac{E}{R_2+G}=K\tan\theta_2$$

$$\text{or}\quad \frac{E}{4+G}=K\tan 45^\circ$$

$$\text{or } \frac{E}{4+G} = K \ldots\ldots\ldots\ldots\ldots\ldots (ii)$$

From the equations (i) and (ii) we get

$$\frac{4+G}{\frac{1}{3}(4+3G)} = \sqrt{3}$$

$$\text{whence } G = \frac{4}{\sqrt{3}} \text{ ohms.}$$

6. A battery is connected to a tangent galvanometer of resistance 9 ohms and produces a deflection of 60° An extra resistance of 7 ohms is then placed in series in the circuit and the deflection falls to 45.° Calculate the resistance of the battery. (Inter. Board. U.P. 1923).

We know that

$$C_1 = \frac{E}{B+G} = K_1 \tan \theta_1$$

$$\text{and } C_2 = \frac{E}{B+G+R} = K_2 \tan \theta_2$$

where C_1 and C_2 are the currents in the two cases. Substituting the given values we get,

$$C_1 = \frac{E}{B+9} = K \tan 60° = K\sqrt{3} \ldots\ldots\ldots (i)$$

$$C_2 = \frac{E}{B+9+7} = K \tan 45° = K \ldots\ldots\ldots (ii)$$

From the equations (i) and (ii) we get,

$$\frac{B+9+7}{B+9}=\sqrt{3}$$

whence B (resistance of the battery)

$$=\frac{16-9\sqrt{3}}{\sqrt{3}-1}$$

$$=0{\cdot}56 \text{ ohm.}$$

7. Calculate the current through a galvanometer of resistance equal to 10 ohms shunted by a resistance equal to 0·1 ohm when the combination forms a part of the circuit, whose total resistance is 10 ohms and in which a cell of e.m.f. of 2 volts is sending the current. [Inter. Board, U.P. 1928].

Let C represent the strength of the main current and C_s and C_g be the currents through the shunt and the galvanometer respectively. We know that

$$C=\frac{E}{R}$$

i.e. $C=\frac{2}{10}=\frac{1}{5}$ ampere.

Again we know that

$$C_g=\frac{s}{s+g}\ C.$$

$$=\frac{0{\cdot}1}{0{\cdot}1+10}\times\frac{1}{5}$$

$$=\frac{0{\cdot}1}{10{\cdot}1}\times\frac{1}{5}$$

$$=\frac{1}{505}\text{ ampere}$$

Exercises.

1. A long vertical wire carries a current of 4 amperes passing downwards. Find the intensity of the magnetic field due to the current at a point 20 cm. from the wire. [Ans. ·04 C. G. S. unit].

2. A vertical wire 62·8 cm. long carries a current of 10 amperes. Find out the intensity of the magnetic field due to the current at a point 20 cm. from the middle of the wire.

Find also the intensity of the field at the centre of a coil made of that wire.

[Ans. 0·084 ; 0·628 C. G. S. unit].

3. A current of 7 amperes passes through a long vertical wire and a magnetic compass needle is kept 10 cm. due (1) east (2) due north of the wire. Find the tangent of the angles through which it would be deflected by the current in each case.

[H = 0·35 gauss].

[Ans. No deflection; $\tan\theta = \frac{2}{5}$].

4. A current of 7·5 amperes flows in a straight vertical wire at a place where the horizontal component of the earth's magnetic field is 0·2 gauss. Compare the times of horizontal oscillations of a small compass needle at two points 10 cm. from the wire (*a*) due north (*b*) due east. (U. L. Inter. 1927).

[Ans. T_a / T_b = 1·183 or 0·447].

5. A coil of 100 turns of wire of a radius equal to 20 cm. carries a current of 5 amperes. Calculate the intensity of the magnetic field at the centre of the coil in absolute units. (Take $\pi = 3{\cdot}14$). [Ans 15·70].

6. Two long straight parallel wires 20 cm. apart, carry currents of 10 amperes. Calculate the strength of magnetic field at a point midway between the wires when currents flow in them (1) in opposite (2) in the same, direction. [Ans. 0·4 gauss ; zero].

7. The same current is sent through two concentric circular coils of radius 2 and 4 decimetres respectively. A small magnetic needle is suspended at their common centre. If the currents flow in opposite directions through the coils, compare their joint effect upon each pole of the needle with that produced by a single coil of 3 decimetres radius traversed by the same current. [Ans. $\frac{3}{4}$].

8. At a distance of 25 cm. from a long vertical wire the intensity of the field is 0·12 gauss. Find the intensity of the field at a place 30 cm. from the wire.

[Ans. 0·1 gauss].

9. A current of 10 amperes passes through a long vertical wire. Find the intensity of the field at a distance of 20 cm. from it. Find also the radius of a coil of two turns which will produce the same inten-

sity at its centre if a current of 2 amperes passes through it. (Take $\pi = 3{\cdot}14$.)

[Ans. $\frac{1}{10}$ gauss ; 25·12 cms.]

10. A small magnet (free to oscillate in a horizontal plane) is placed 10 cm. due magnetic west of a long vertical wire and is found to make 12 oscillations per minute under the influence of the earth's horizontal field (0·36 gauss.) How many oscillations per minute will it make when a current of 6·5 amperes runs (1) downwards, (2) upwards, in the wire ?

[Ans. 14 ; 9·59.].

11. A tangent galvanometer having a coil of one turn of 34 centimetres radius gives a deflection of 45° with a current of 10 amperes. Calculate the strength of the earth's magnetic field at the centre of the coil. (Lond. Univ. Inter. B. Sc. 1907)

[Ans. 0·18 (nearly)]

12. The coil of a galvanometer consists of eight turns of wire, and has a mean radius of 20 centimetres. Find what current will produce a deflection of 45° if the horizontal intensity of the earth's magnetic field is 0·18 C.G. S. unit.

(Lond. Univ. Matric, 1903)

[Ans. 0·716 ampere]

13. A tangent galvanometer consists of 7 turns of wire, each of 44 cm. diameter. A current of 1·8 amperes is allowed to flow through it. Find the strength of the field due to the circular current at its centre. If the value of H at the place is 0·36, through what angle will the needle be deflected?

[Ans 0·36; 45°]

14. A small magnet is suspended at the common centre of two concentric circular wires of 4 and 6 decimetres diameter respectively. The same current is sent through the wires but in opposite direction. Show that $\tan\theta_1 : \tan\theta_2 = 5 : 12$, where θ_1 and θ_2 are the deflections in the two cases.

15. Calculate the strength of a current required to produce a deflection of 45° in a tangent galvanometer which consists of a single turn of thick wire of 30 cm. radius. H is given to be 0·36 gauss.

$\left(\text{Take } \pi = \frac{22}{7}\right)$ [Ans. 17·18 amp.]

16. Find the intensity of the magnetic field at the centre of the coil of a tangent galvanometer consisting of 20 turns of wire of 44 cm. diameter, through which passes a current of 2 amperes. Taking H=0·36 gauss calculate the reduction factor of the galvanometer.

[Ans. $1\frac{1}{7}$ units; 0·063 for C.G.S. units.]

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17. A circuit of total resistance 100 ohms includes a galvanometer of 5 ohms resistance. The galvanometer is shunted with 5—ohm coil. Find the ratio of the current in the galvanometer before and after it is shunted. [Ans. 39 : 20.]

18. Two cells, the E M. F's of which are 2 : 1 are joined up in series, with their E. M F's acting in the same direction, and the circuit is completed through a tangent galvanometer, the needle of which is deflected through 60°. If one of the cells is reversed, no other change being made, what will be the deflection of the galvanometer? (B. of E. 1896)

[Ans. 30°].

19. A current flows through two tangent galvanometers in series, each of which consists of a single ring of copper, the radius of one ring being three times that of the other. In which of the galvanometers will the deflection of the needle be greater? If the greater deflection be 60°, what will the smaller be?

(B. of E. 1897) [Ans. 30°]

20. Two tangent galvanometers, A and B, are identical in construction, except for the number of turns in the coil. They are connected in series and a current is sent through them. The deflection in A is 45°, and that in B is 31°. Calculate the ratio of the

numbers of turns in the two instruments. (Tangent $31^\circ = 0·60$) [L. U.] (Ans. $n_a : n_b = 5 : 3$).

21. A current flowing through a tangent galvanometer consisting of 10 turns of wire of radius 8 cm. produces a deflection of 45° when the instrument is in a position where H = 0·18 dyne per unit pole. What alterations would you make in the instrument so that it would give this same deflection for a current of one thousandth of an ampere? (L. U.)

[Ans. Increase number of turns to 2292].

22. A wire AB of 0·33 ohm resistance forms part of a circuit through which an electric current flows in the direction from A to B. The points A and B are also connected by another conducting path in which is included a cell of E. M. F. 1·287 volts and a galvanometer, the positive pole of the cell being that joined to A. If the galvanometer is not deflected, what is the strength of the current in wire AB?

(B. of E. 1897) [Ans. 3·9 amperes].

23. The coil of a tangent galvanometer is placed at right angles to the magnetic meridian and a steady current passes through it. The needle when set in vibration makes 5 oscillations in a given time, but only 3 in the same time when the direction of the current is reversed. Compare the magnetic

force at the centre of the coil due to the current with that due to the earth. S and A. D. (A) 1898.

$\left[\text{Ans.} \frac{8}{17}H\right]$

24. A battery is connected to a tangent galvanometer of resistance 9 ohms and produces a deflection of 60°. An extra resistance of 7 ohms is then placed in the circuit and the deflection falls to 45.° Calculate the resistance of the battery (Sen. Camb. Loc.)

[Ans. 0·5628 ohm]

25 Two voltaic cells A and B are connected in series, and form a simple circuit with a galvanometer. The current indicated is 2·4 amperes. The cell A is then reversed so as to oppose B, and the current observed is 0·6 ampere in the same direction as before. Calculate the ratio of the electromotive forces of A and B. (L. U.)

[Ans. $E_A : E_B = 3 : 5$]

26. Two cells, when connected in series, give rise to a deflection of 45° on being joined to the terminals of a tangent galvanometer, and to a deflection of 30° when connected so as to oppose each other. Compare the electromotive forces of the cells. (L. U.)

[Ans. 3·733 : 1]

27. A Daniell cell connected to a tangent galvanometer of 0·5 ohm resistance produces a deflection of 60°. On interposing a resistance of 2 ohms the deflection falls to 30°. What is the internal resistance of the cell? (U. L. 1906) [Ans. 0·5 ohm].

28. A battery sends a current through a tangent galvanometer, a resistance box and a coil, in series. When a resistance of 7 ohms is taken from the box, the deflection is 40°. The coil is then removed and a resistance of 18 ohms is introduced in the box to produce the same deflection. Calculate the resistance of the coil. [Ans. 11 ohms].

29. A battery (of negligible resistance), a tangent galvanometer, a resistance box and an unknown coil are all connected in series. The deflection in the galvanometer with 2 ohms resistance in the box is 56°. The unknown coil is then removed and 20 and 10 ohms are introduced in the box when the deflections produced are 50° and 60° respectively. Calculate the resistance of (1) the galvanometer (2) the unknown coil.

[tan 50° = 1·19 ; tan 56° = 1·48 ; tan 60° = 1·73].

[Ans. (1) G = 12·04 ohms; (2) 11·72 ohms].

30. A battery, a tangent galvanometer and a resistance of 4 ohms are connected in series. The deflection of the needle is 50°. On increasing the resistance to 6 ohms, the deflection falls to 40°.

Calculate the resistance of the battery if the resistance of the galvanometer is given to be 0·12 ohm. [tan 50° = 1·19 ; tan 40° = 0·84.]

[Ans. 0·68 ohm].

31. A current from a battery is sent through a resistance box and a tangent galvanometer of 0·25 ohm resistance. With a resistance of 50 ohms, a deflection of 60° is obtained. When the resistance box is replaced by a wire of unknown resistance, a deflection of 30° is obtained. Calculate the value of the unknown resistance. [The resistance of the battery = ·1 ohm; tan 60° = 1·73 ; tan 30° = ·58.]

[Ans. 149·83 ohms].

32. A battery (of negligible resistance) is connected with a tangent galvanometer of resistance 200 ohms, and a wire of unknown resistance in series. A deflection of 45° is produced: on increasing the resistance of the circuit by 50 ohms, the deflection is reduced to 40°. Calculate the resistance of the wire. [tan 40° = ·84]

[Ans. 62·5 ohms].

33. A cell of e.m.f. 1·3 volts is in series with a tangent galvanometer and a resistance. The latter is so adjusted that a deflection of 42° is obtained. When an additional resistance of 5 ohms is introduced, the deflection is reduced to 40°. Calculate the reduction factor of the galvanometer.

cot $40^\circ = 1\cdot19$; cot $42^\circ = 1\cdot11$; tan $40^\circ = \cdot84$; tan $42^\circ = \cdot90$.

[Ans. 0·021].

34. A cell is connected in series with a resistance box and a tangent galvanometer. A certain convenient resistance is introduced in the box when the deflection is 60°. The first cell is now replaced by a second when a deflection of 48° is obtained with the same resistance. Compare the E.M.F's of the two cells. Battery resistances are negligible.

tan $60^\circ = 1\cdot73$; tan $48^\circ = 1\cdot11$.

$$\left[\text{Ans } \frac{E_1}{E_2} = \frac{1\cdot56}{1}\right]$$

35. Two cells of E.M F's E_1 and E_2 are connected in series and are then used in sending a current through a tangent galvanometer and a resistance box. A deflection of 44° is obtained with a suitable resistance plugged from the box. The cells are then connected so as to oppose each other and a deflection of 30° is obtained with the same resistance. Compare the E.M. F's of the two cells.

tan $44^\circ = 0\cdot97$; tan $30^\circ = 0\cdot58$,

[Ans $E_1 : E_2 = 3\cdot97 : 1$.]

36. The galvanometer whose resistance is 300 ohms is so shunted that only 0·01 of the total current flows in it. What is the resistance of (*a*) the shunt,

and (*b*) of the galvanometer and shunt combined? (Allahabad University.)

[Ans. 3·03 ohms; 3 ohms].

37. A circuit consists of a battery of resistance B and a galvanometer of resistance G. If the galvanometer is shunted with a resistance S, how must the resistance in the circuit be altered to keep the battery current unchanged? (U.L.)

$$\left(\text{Ans. } \frac{G^2}{G+S} \text{ increase.}\right)$$

38. A current of ½ ampere passes through a tangent galvanometer whose resistance is 10·5 ohms. After the terminals of the galvanometer have been joined by a wire, the total current in the circuit remains unaltered but the current in the galvanometer is reduced to $\frac{1}{20}$ ampere. If the resistance of the wire is 14 ohms per metre, what length of wire has been used as a shunt? (Oxford Local. Senior. 1902.)

[Ans. 0·083 metre].

39. The resistance of a galvanometer is 100 ohms; it is connected in circuit with a battery, whose E.M.F. is 15 volts; the resistance of the battery and wires is 5 ohms; the galvanometer is shunted so that one-tenth of the total current passes through

it. Find the resistance of the shunt, and the current through the battery. (Camb. Local. Senior. 1884).

[Ans. $\frac{100}{9}$ ohms ; 1 ampere.]

40. The galvanometer of 45 ohms resistance is shunted by a shunt of 5 ohms. Find the resistance of the shunted galvanometer and the current which flows through it when a difference of potential of 22·5 volts is maintained between its terminals. (Inter. Board U. P. 1920).

[Ans. 4·5 ohms; 0·5 ampere].

41. A galvanometer having a resistance of 40 ohms gives a deflection of one scale division for a current of $\frac{1}{1001}$ ampere. Find the magnitude of the resistance required, and show how it must be connected, to change the galvanometer into : (*a*) an ammeter reading 1 ampere per scale division, (*b*) a voltmeter reading 1 volt per scale division, (B.U.)

[Ans. (*a*) 0·04 ohm in parallel with G.

(*b*) 961 ohms in series with G]

42. The resistance between the terminals of an ammeter is $\frac{1}{10}$ ohm and its range from 0 to 5 amperes. Determine the resistance of the shunt necessary to fit

it for the measurement of currents from 0 to 50 amperes. (U. L.)

(Ans. $\frac{1}{90}$ ohm.)

43. A galvanometer connected (*a*) in series, (*b*) in parallel, with a resistance of 3 ohms and a battery of constant electromotive force and negligible resistance, indicates currents which are in the ratio of 3 to 4. Find the resistance of the galvanometer.

(Camb. Local. Senior. 1898.) [Ans. 9 ohms].

44. A length of a potentiometer wire of 188 cm. balances the e. m. f of a cell on open circuit, and a length of 168 cm. when the cell has a resistance equal to 12 ohms, connected between its terminals. Calculate the internal resistance of the cell.

[Ans. 1·428 ohms].

45. A length of a potentiometer wire of 195 cm. balances the e. m. f. of a cell of internal resistance 1 2 ohms when the circuit is open. What length of the potemtiometer wire would balance the e. m. f. of the cell when a resistance of 5 ohms is placed across its terminals ?

[Ans 158·1 cm.]

End

miss Tupoo.

CHAPTER V.

Electrolysis.

The following are the two laws of electrolysis due to Faraday :—

(1) The mass of an element liberated during the process of electrolysis in a cell or voltameter in a given time is proportional to the total quantity of electricity passed in that time. Thus if m denotes the mass of an element liberated by a current of C amperes in t seconds, then—

$m \propto Ct = Czt$, where z is a constant known as electro-chemical equivalent for that particular element.

(2) If the same current be passed through a number of cells or voltameters arranged in series the masses of ions liberated are proportional to their "chemical equivalents". (Where chemical equivalent $= \dfrac{\text{atomic weight}}{\text{valency}}$.)

Thus if C amperes of current be passed through Copper, Silver, and Water voltameters connected in series it will be found that when 1 gram of hydrogen is liberated $\frac{63}{2}$ or 31·5 grams of copper, $\frac{108}{1}$ or 108

grams of silver, $\frac{16}{2}$ or 8 grams of oxygen will be liberated in the voltameters.

Electro-chemical equivalent of an element :— It is equal to the mass of an element set free by electrolysis by a current of one ampere in one second *i e.* by one coulomb of electricity. Thus it follows from what has already been said that electro-chemical equivalent of an element is proportional to its chemical equivalent.

Unit :—gms/coulomb.

Solved Examples.

1. A metal plate having a surface of 100 sq. cm. is plated with silver 0·019 mm. thick. To deposit this silver a current of 2·5 amperes is used for 12 minutes. Calculate the electro-chemical equivalent of silver. Density of silver = 10·6 gm. per c. c.

[Inter. Board. U. P. 1933].

Volume of silver deposited

$$= 100 \times \frac{\cdot 019}{10} \text{ cubic cm.}$$

$$= 0 \cdot 19 \text{ c. c.}$$

∴ Mass of silver deposited

$$= 0 \cdot 19 \times 10 \cdot 6 \quad \left[D = \frac{m}{V} \right]$$

$$= 2 \cdot 014 \text{ gms.}$$

But we know that

$$m = C z t.$$

$$\therefore z = \frac{m}{Ct}$$

Substituting the given values we get,

$$z = \frac{2 \cdot 014}{2 \cdot 5 \times 12 \times 60}$$

$$= \cdot 00111\dot{8}$$

Hence the electro-chemical equivalent of silver $= 0 \cdot 00111\dot{8}$ gms/coulomb.

2. Find the strength of the current which will deposit 2 gms. of silver in 20 minutes. [I. B. 1931].

We know that

$$m = C z t.$$

$$\therefore \quad C = \frac{m}{zt}$$

Substituting the given values we get,

$$C = \frac{2}{0\,00\,1118 \times 20 \times 60} \; (\because z \text{ of silver} = \cdot 001118)$$

$$= 1\cdot 5 \text{ amperes.}$$

Hence the required current is 1·5 amperes.

3. A piece of metal weighing 200 grams is to be electroplated with 2·5 per cent of its weight by gold. If the current is 1 ampere, $z = \cdot 0006808$ gm. per coulomb, how long will it take to deposit the required weight of gold ? [I. B. 1927].

Mass of gold to be depsited on the metal piece.

$$= \frac{200 \times 2\cdot 5}{100} = 5 \text{ gms.}$$

We know that $m = C z t.$

$$\therefore t = \frac{m}{Cz}$$

Substituting the given values we get,

$$t = \frac{5}{1 \times 0\cdot 0006808.}$$

$$= \frac{5 \times 10^7}{6808} \text{ secs.}$$

$$= \frac{15625}{7659} \text{ hours.}$$

$$= 2 \text{ hrs. } 2 \text{ mins. } 24{\cdot}3 \text{ secs.}$$

Hence the required time is 2 hrs. 2 mins. 24·3 secs.

4. A constant current is passed for 20 mins. through a silver voltameter and a tangent galvanometer connected in series. The weight of silver deposited is found to be 0·2 gm. Deflection in the galvanometer is found to be 45°. Calculate K when z = ·00112. [Inter. Board U. P. 1926.]

We know that in the case of a tangent galvanometer

$$C = K \tan \theta.$$

But $$m = C z t.$$

$$= K \tan \theta . z . t.$$

Substituting the giveu values get,

$$0{\cdot}2 = K. \tan 45^\circ \times {\cdot}00112 \times 20 \times 60.$$

$$= K \times 1 \times {\cdot}00112 \times 20 \times 60.$$

(Since $\tan 45^\circ = 1$)

$$\therefore K = \frac{{\cdot}2}{{\cdot}00112 \times 20 \times 60}$$

i.e. $K = 0{\cdot}149.$

Hence the required value of.

$K = 0{\cdot}149$ ampere.

5. It is found that in 1 min 40 seconds a certain current deposits 0·112 gm. of silver, and in twice the time 0·081 gm. of potassium. Given that the chemical equivalent of silver is 108, find that of potassium. (U. L.)

The mass of silver deposited in double the given time.

$= 2 \times 0{\cdot}112.$

$= 0{\cdot}224$ gm.

During the same time, mass of potassium deposited is given to be 0·081 gm.

But we know from Faraday's second law of electrolysis that the masses of different substances liberated by a given current in a given time are proportional to their chemical equivalents.

$$i.\ e.\ \frac{m}{m'} = \frac{\text{Chemical equivalent of silver}}{\text{Chemical equivalent of potassium}}.$$

$$\text{Or}\quad \frac{0{\cdot}224}{0{\cdot}081} = \frac{108}{\text{Chemical equivalent of potassium.}}$$

Hence the required chemical equivalent of potassium

$$= \frac{108 \times {\cdot}081}{{\cdot}224}$$

$$= 39{\cdot}05.$$

6. 5 Cells, each of E. M. F. 2 volts and resistance 0·04 ohm, are arranged in series, and drive a current between platinum electrodes immersed in dilute sulphuric acid. The resistance of the acid between the electrodes is 4 ohms, and the E. M. F. of polarisation is 1·5 volts. Calculate the mass of water decomposed in an hour. Electro-chemical equivalent of hydrogen is 10^{-5} gms. / coulomb. (U. L.)

The effective E. M. F. of the battery

$$= 2 \times 5 - 1{\cdot}5 = 8{\cdot}5 \text{ volts.}$$

(Since the E. M. F. of polarisation or back E.M F. is given to be 1·5 volts.)

Total resistance of the circuit.

$$= 5 \times 0{\cdot}04 + 4.$$
$$= 4{\cdot}2 \text{ ohms.}$$

Hence $C = \dfrac{E}{R}.$

$$= \frac{8{\cdot}5}{4{\cdot}2} \text{ amperes.}$$

Let m be the mass of hydrogen liberated by this current.

Then $m = Czt$

$$= \frac{8{\cdot}5}{4{\cdot}2} \times 10^{-5} \times 60 \times 60$$

$$= \frac{51}{700} \text{ gm.}$$

But we know that 2 gms. of hydrogen are liberated when 18 gms. of water are decomposed.

Hence the required mass of water decomposed

$$= \frac{51}{700} \times \frac{18}{2}$$

$$= 0{\cdot}656 \text{ gms.}$$

7. A current which passes through a standard resistance of 2 ohms also passes through a copper voltameter in which 0·85 gm. of copper is deposited in 25 minutes. If a voltmeter of very high resistance connected to the ends of the standard resistance reads 3·5 volts while the current is flowing, what is the error of the voltmeter?

[E. Ch. E. of copper = 0·000326.] (U. L.)

We know that $m = C z t$.

$$\therefore C = \frac{m}{z t}$$

Substituting the given values we get,

$$C = \frac{\cdot 85}{\cdot 000326 \times 25 \times 60}$$

$$= 1{\cdot}738 \text{ amperes.}$$

This is the current passing through the voltameter and the standard resistance of 2 ohms. Applying ohm's law in the standard resistance circuit we get the potential difference between its terminals.

$$i.e.\ E = C \times R.$$

$$= 1{\cdot}738 \times 2$$

$$= 3{\cdot}476 \text{ volts.}$$

But the voltmeter reads 3·5 volts,

Hence the error in the voltmeter.

$$= -\ 0{\cdot}024 \text{ volt.}$$

8. A steady current passing for 10 minutes through a dilute solution of acid liberates 100 c. c. of oxygen measured at 27° C. and 740 mm. pressure. If the electro-chemical equivalent of hydrogen is 0·0000104 gm. per coulomb and one litre of hydrogen at N. T. P. weighs 0·089 gm., find the current. (U. L.)

Let us first find out the volume of oxygen liberated, at N. T. P.

We know that $\dfrac{P_0\, V_0}{T_0} = \dfrac{P_1\, V_1}{T_1}$

$$\therefore\ V_0 = \frac{P_1\, V_1}{T_1} \times \frac{T_0}{P_0}$$

Substituting the given values we get the volume of oxygen at N. T. P.

$$i.e.\ V_0 = \frac{740 \times 100}{300} \times \frac{273}{760} = \frac{37 \times 91}{38} \text{ c.c.}$$

1 litre of hydrogen at N. T. P. weighs 0·089 gm.

$\therefore$ 1 litre of oxygen at N. T. P. will weigh

$$16 \times 0{\cdot}089 \text{ gm.}$$

Hence $\left(\frac{37 \times 91}{38}\right)$ c.c. of oxygen at N. T. P.

will weigh. $\frac{16 \times 0{\cdot}089 \times 37 \times 91}{1000 \times 38}$ gm.

$$= \frac{16 \times 89 \times 37 \times 91}{38 \times 10^6} \text{ gm.}$$

We know that the electro chemical equivalent of an element is proportional to its chemical equivalent.

$$\text{Hence } \frac{\text{E. Ch. E of Oxygen}}{\text{E. Ch E. of Hydrogen}} = \frac{\frac{16}{2}}{1}.$$

$\therefore$ E. Ch. E. of Oxygen $= 8 \times$ E. Ch. of Hydrogen

$$= 8 \times {\cdot}0000104$$

$$= {\cdot}0000832.$$

Again $\quad m = Czt$

$$\therefore \quad C = \frac{m}{zt}$$

$$= \frac{\frac{16 \times 89 \times 37 \times 91}{38 \times 10^6}}{{\cdot}0000832 \times 10 \times 60}$$

$$= \frac{16 \times 89 \times 37 \times 91}{38 \times 10^6 \times {\cdot}0000832 \times 600}$$

$$= 2{\cdot}53 \text{ amperes.}$$

Exercises.

1. A current of 0·75 ampere flows between two plates of copper immersed in a solution of copper sulphate. What changes take place in 5 mins?

[E. Ch. E. of H_2 = ·0000104.] [Inter. Board. U. P. 1918].

[Ans. 0·074 gm.]

2. How many amperes would deposit 10 grams of copper in 1 hour 15 mins. the current being supposed constant? [E. Ch. E. of Cu = ·00033].

[Ans. 6·73 amperes].

3. How many grams of copper would be deposited by a constant current of 6 amperes acting for 30 mins.?

[Ans. 3·564 gms.]

4. What would be the strength of a constant current which would liberate 100 cubic centimetres of hydrogen in 5 minutes? [1 litre of H_2 at N. T. P. weighs 0·089 gm.]

[Ans 2·85 amperes].

5. If 3 amperes deposit 4 grams of silver in 20 minutes, what is the electro-chemical equivalent of silver? (B. of E. 1899].

[Ans. 0·001.].

6. A current of 1 ampere is passed for 2 hours through an electrolyte and decomposes 2·4 grams. Find the electro-chemical equivalent of the electrolyte.

[B of E 1905]. [Ans. 0·0003].

7. A battery of 6 Daniell cells in series sends a current through a solution of silver nitrate. Find the amount of zinc dissolved in each cell while 1 gm. of silver is deposited. (At. wt. of silver = 108, of zinc = 65). (U. L.).

[Ans. 0·30 gram.]

8. If the electro-chemical equivalent of silver is ·01118, what is the electro-chemical equivalent of oxygen? (B. E.)

[Ans. 0·000828]

9. If 9 amperes deposit 10 grams of silver in 16 minutes, how much copper would 16 amperes deposit in 9 minutes?

[Atomic weights: silver 108, copper 63·5; valencies: silver 1, copper 2]. (U L.)

[Ans. 2·94 gms.]

10. What weight of hydrogen is separated from water by the passage of 1000 coulombs of electricity, given that the chemical equivalent of copper is 31·5, and its electro-chemical equivalent 0·000328 per coulomb? [B. of E. 1902].

[Ans. 0·0104 gram].

11. How many amperes would liberate 500 c. c. of hydrogen in 31 min. 4 sec., the current being constant ?

[E. Ch. E. of H_2 = 0·0000104 ; 1 litre of H_2 at N. T. P. weighs 0·089 gm] [Ans. 2·3 amperes].

12. If a given current can deposit an ounce of silver in a given time, what weight of water will it decompose in the same time, and what weight of copper will it deposit ? The atomic weights are $H' = 1$, $O'' = 16$, $Cu'' = 63·5$, $Ag' = 108$.

[Matric. L. U. 1892].

[Ans. 0·083 oz. of water ; 0·291 oz. of copper].

13. Given that a coulomb will electrolyse $9·4687 \times 10^{-5}$ grams of water, what current (in amperes) will be required to electrolyse 0·4 gram of water in one hour ? (Oxf, Local. Senior. 1908).

[Ans. 1·17].

14. A tangent galvanometer was joined in series with a battery and a silver voltameter. The deflection of the needle was 45°, and in the course of an hour the mass of silver deposited was 0·1052 gram. Given that the electro-chemical equivalent of silver is 0·001118, calculate the constant of the galvanometer.

(L. U.) [Ans. 0·0261.]

15. A tangent galvanometer has a current passed through it which deflects it 45°. The same current passes through a copper voltameter, where it deposits 0·3 gm. of copper in 30 minutes. If the electro-chemical equivalent of copper is 0.00033 gram/ampere-second, find the value of the current, and show how to determine the current for any other reading of the galvanometer. (U. L.)

[Ans. 0·505 ampere, $C = K \tan\theta = \cdot 505 \times \tan\theta$].

16. A tangent galvanometer having a coil 12 turns and diameter 18 cm. is placed in series with a copper voltameter. If the current flows for 1 hour and 0·21 gm. of copper is deposited, the deflection all the time being 30°, what is the value of the earth's magnetic field ?

[E. Ch. E of copper = 0·00033.]

[Ans. 0·257 gauss].

17. A current flowing through a tangent galvanometer and a copper voltameter in series produces a deflection of 60° and deposits 0·522 gm. of copper in 45 minutes. If the diameter of the coil is 18 cm., H = ·019 gauss, and the electro-chemical equivalent of silver is 0·001118 gm. per coulomb, find the number of turns in the coil of the galvanometer, taking the atomic weight of silver as 108 and of copper 63·6. (L. U.). (Ans. 8·017.)

18. A circuit includes a silver voltameter and a tangent galvanometer of 20 turns of 16 cm. diameter. If the galvanometer shows a steady deflection of 45° and if 0·115 gm. of silver is deposited in 15 minutes, find the strength of the earth's horizontal magnetic field.

[Electro-chemical equivalent of silver = ·001118 gm. per coulomb] (L. U.)

[Ans. 0·1795 C. G. S. unit).

19. A copper voltameter is connected in series with a tangent galvanometer having 24 turns of 30 cm. diameter. If the deflection is 60° and H = 0·36, calculate the amount of copper deposited in 30 min.

[E. Ch. E. of Cu. = ·00033].

[Ans. 0·368 gm.]

20. How much electricity will be required to liberate a cubic-metre of dry hydrogen at 15° C. ?

Assume electro-chemical equivalent of copper = 0·000328, atomic weights Cu = 63·1; H = 1·008, density of Hydrogen at N. T. P. = 9×10^{-5}]. (U. L.)

[Ans. $8·14 \times 10^{6}$ coulombs].

21. A copper voltameter is connected in series with a wire having a resistance of 2·65 ohms for a length of 122 cm. What weight of copper will be

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deposited in half an hour if the potential gradient along the wire is 1 volt per metre ?

[Electro-chemical equivalent of copper = 0·000329 gram per coulomb]. (U. L.).

[Ans. 0·273 gm.]

22. An ammeter reads 0·93 ampere and the weight of copper deposited in a copper voltameter in series with it is 1·602 gm. The current flows for 1 hr. 30 min. Find the correction to be applied to the ammeter reading.

[E. Ch. E of copper = 0·00033].

[Ans. − 0·0311].

23. The volume of hydrogen liberated from a water voltameter is 860 c.cm. at 68 cm. pressure and 15° C. A current of 1·2 amperes as read by an ammeter was allowed to pass for 1 hour 30 mins. Find the error in the ammeter reading.

[E. Ch. E. of oxygen = 0·0000829 3, atomic wt. of oxygen = 16, valency = 2. One litre of hydrogen at N. T. P. weighs 0·089 gm]. [Ans. − 0·04].

24. If a current of 0·5 ampere is passed for 10 minutes between platinum electrodes immersed in acidulated water, what weight of hydrogen is evolved, and what volume would it occupy at 0° C and 760 mm.

pressure ? What weight of oxygen is given off at the same time ?

The electro-chemical equivalent of hydrogen is 0·0000104 gm. per coulomb ; that of oxygen is 8 times as great.

Hydrogen, at 0°C and 760 mm. pressure weighs 0·00009 gm. per c. c.]

(Civil Service Commission).

[Ans. 0·00312 gm ; 34·67 c. c ; 0·025 gm.]

CHAPTER VI

HEATING EFFECTS OF CURRENT.

If a quantity of electricity Q units be sent through a portion of circuit, at the ends of which a potential difference of E units is maintained, the amount of electrical energy expended is measured by the product of the quantity of electricity and the difference of potential *i. e.* QE. But the quantity of electricity Q is given by the product of the current and the time for which the current flows *i.e.* Q = Ct, hence the work done for the passage of Q units of electricity is given by

$$W = QE = ECt \ldots\ldots\ldots\ldots (1)$$

Electric power :—The rate at which work is done is the measure of the 'electric power' expended in the circuit. This is proportional to the product of the Current and E. M. F. and is independent of the time during which the current flows.

$$i.\ e.\ P = \frac{QE}{t} = EC \ldots\ldots\ldots\ldots\ldots (2)$$

If both C and E are expressed in C. G. S. units the work is measured in ergs and the power in ergs per second. If C is in amperes and E in volts, the rate at which the current does work $= 10^{-1} \times 10^8 = 10^7$ ergs per second (since 1 ampere $= 10^{-1}$ C. G. S. units and volt $= 10^8$ C. G. S. units of E. M. F.) This

is taken as the practical unit of power and is termed a '**watt**'.

Hence, the **Rate of Working** (expressed in watts) = **Amperes** × **Volts.** **Watt** is defined as the rate of working necessary to supply energy at the rate *one Joule per second.* (1 Joule = 10^7 ergs).

Another unit of power frequently used in engineering is called the **Horse-power.**

1 H. P. = 746 watts. (See Willow's Text book of Physics p. 407).

In the distribution of power by electricity another still larger unit for the rate of working or the electric power is used and is called the **Kilowatt** and is equal to 1000 watts. The unit of energy consumed or the electrical work done is the **Kilowatt-hour,** which is the amount of work done when a rate of working of *1 kilowatt is continued for 1 hour.* This is also known as the **Board of Trade Unit.** Thus a (B. O. T.) = $3{\cdot}6 \times 10^6$ Joules.

The following laws relating to the generation of heat in a conductor due to the passage of electricity were established emperically by Joule :—

The heat developed in a circuit by a current is found to be

(1) proportional to the square of the current strength,

(2) proportional to the resistance, and.

(3) proportional to the time for which the current flows.

i.e. **Heat produced** $\propto C^2 Rt$.

If E volts denote the P. D. at the ends of a resistance and C the current in amperes flowing through it for t secs., the total electrical energy expended is given by $E. C. t \times 10^7$ ergs.

If all this electrical energy is converted to heat, the **Heat** developed in the resistance is expressed by the following equation :—

$$H \text{ (in calories)} = \frac{E. C. t \times 10^7}{J}$$ where J is the mechanical equivalent of heat. ($J = 4{\cdot}2 \times 10^7$ ergs per second).

Hence, **Heat in Calories**

$$= \frac{ECt \times 10^7}{4{\cdot}2 \times 10^7}$$

$$= \frac{1}{4.2} E.C.\ t.$$

$$= 0{\cdot}24\ E.\ C.\ t. \ldots\ldots (1)$$

$$= 0{\cdot}24\ \frac{E^2 t}{R} \ldots\ldots\ (2) \quad (\because C = \frac{E}{R} \text{ by ohm's law}).$$

$$= 0{\cdot}24\ C^2 Rt \ldots\ldots (3)$$

Glass Bulbs:—

On the cap of the electric bulbs are marked figures which indicate the particular circuit in which they can be used for safety and the total electrical energy consumed by each per hour. For example a bulb having 220—40W marked on it means that the lamp is made for use on a 220-volt circuit and that its rate of consumption of power is 40 watt hours or ($40 \times 60 \times 60$ Joules per sec.) or ($40 \times 60 \times 60 \times 10^7$ ergs per sec).

Efficiency:—

Efficiency of a lamp is the power consumed for each candle-power of light emitted and is expressed in watts per candle-power. If a 40—watt lamp gives 25 candle-power, the efficiency is $\frac{40}{25} = 1{\cdot}6$ watts per candle-power.

Half-watt lamps :—

As the name implies **Half-watt** lamps are those lamps which have a consumption of nearly 0·5 watt only per each candle-power of light given.

A 40 c. p. half-watt lamp means that its rate of consumption is 20 watts per hour. The filaments of such lamps is of **Tungsten** and is closely wound and the globe is filled with nitrogen at about two-thirds the atmospheric pressure. The chief advantages of these lamps are high efficiency and longer life. The term half-watt has now been replaced by the term **'Gas filled'**.

Solved Examples.

1. A fine wire is placed in 500 grams of water in a light copper calorimeter, and 5 amperes are passed through it, an E. M. F. of 14 volts being observed between the ends of the wire. Calculate the rise of temperature in 10 minutes. [A water-gram-degree centigrade $= 4 \cdot 2 \times 10^7$ ergs].

We know that heat produced in a conductor is given by

$$\text{H (in calories)} = \frac{\text{E. C. t} \times 10^7}{\text{J}}$$

(where $\text{J} = 4 \cdot 2 \times 10^7$ ergs).

$$= \frac{14 \times 5 \times 1 \times 10^7}{4 \cdot 2 \times 10^7} \text{ (in one second)}$$

Hence the heat produced in 10 minutes

$$= \frac{14 \times 5 \times 10 \times 60}{4 \cdot 2} = 10000 \text{ Calories.}$$

Let the temperature of the given mass of water be raised through t°C.

$\therefore$ Heat taken by water $= m \times s \times t$.

$$= 500 \times t.$$

Hence we have

$$500\,t = 10000$$

$$\therefore\ t = 20^\circ\text{C.}$$

2. A Daniell's cell has an internal resistance of 2 ohms. Compare the amounts of heat produced in the cell for each gram of zinc consumed in the battery (1) when the cell is short-circuited, (2) when the terminals are connected by a resistance of 2 ohms, (3) when they are connected by 100 ohms. (U. L.)

The amount of zinc consumed in the cell in all the three cases is 1 gram and therefore the quantity of electricity ($Q = Ct$) flowing through the cell will be the same in all the cases.

We know by Joule's law that heat developed in a conductor is proportional to C^2Rt (where R is the resistance of the conductor).

If H_1, H_2 and H_3 are the heats produced (in the three cases and C_1, C_2 and C_3, the respective currents flowing through the battery we have,

$H_1 : H_2 : H_3 = C_1^2R\,t_1 : C_2^2Rt_2 : C_3^2Rt_3$ (where R represents the internal resistance of the battery)

$$= C_1 : C_2 : C_3 \quad (\text{Since } C_1t_1 = C_2t_2 = C_3t_3 = \text{a constant}).$$

Now $C_1 = \dfrac{E}{2}$ (since the external resistance is zero).

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$$C_2 = \frac{E}{2+2} = \frac{E}{4}$$

and $$C_3 = \frac{E}{2+100} = \frac{E}{102}$$

Hence $$H_1 : H_2 : H_3 = \frac{E}{2} : \frac{E}{4} : \frac{E}{102}$$

$$= 1 : \tfrac{1}{2} : \frac{1}{51}.$$

3. On passing a current of 1 ampere through a piece of platinum wire, it is found that its temperature rises 10°C. above that of the surrounding objects, which are at 0°C. Assuming that the rate of loss of heat is proportional to the difference of temperature, calculate the temperature of the wire when a current of 2 amperes is passed through it. The temperature coefficient of the resistance of the wire may be taken to be ·004 of the resistance at 0°C. (L. U.)

Suppose the temperatures of the wire in the two cases are T_1° and T_2°C, (assuming no loss of heat due to radiation.).

The rate of loss of heat is given to be proportional to the difference of temperature and so we shall have,

$$t' = (T_1 - k\,T_1) \ldots\ldots\ldots (1)$$

and $$t'' = (T_2 - k\,T_2) \ldots\ldots\ldots (2)$$

Where t' and t'' are the actual temperatures (after radiation) and k, a constant.

Dividing (1) by (2) we get,

$$\frac{t'}{t''} = \frac{T_1\ (1-k)}{T_2\ (1-k)} = \frac{T_1}{T_2} \ldots\ldots\ldots(3)$$

We know that the heat produced in a conductor is given by C^2 Rt.

Hence in the first case the heat produced per second in the wire is given by

$$H_1 = (1)^2 . R_{t'} = (1)^2 . R_0\ (1 + \alpha t'), \ldots.(4),$$

where α is the temperature coefficient.

Similarly in the second case we shall have,

$$H_2 = (2)^2\ R_{t''} = 4 . R_0\ (1 + \alpha t'') \ldots\ldots..(5).$$

The temperatures to which the wire will be raised in the two cases will be $\propto$ to the rate of heat generated (the wire being the same).

$$\therefore \frac{H_1}{H_2} = \frac{1}{4} . \frac{(1+\alpha t')}{(1+\alpha t'')} = \frac{T_1}{T_2} = \frac{t'}{t''} = \frac{10}{t''}$$

$$\text{Or}\quad \frac{1 + \cdot 004 \times 10}{4(1 + \cdot 004 t'')} = \frac{10}{t''}$$

Whence $t'' = 45 \cdot 45°C$.

4. What current will be required by a lamp marked 225V, 40W ? Find the resistance of the filament of such a lamp. (Inter. Board. U. P. 1932).

We know that,

Watt = E × C

$$\therefore C = \frac{\text{Watt}}{E} = \frac{40}{225} = \frac{8}{45} \text{ ampere.}$$

Hence the required current $= \frac{8}{45}$ ampere.

∴ Resistance of the filament of the lamp

$$= \frac{E}{C} = \frac{225}{\frac{8}{45}} = 1265 \cdot 6 \text{ ohms.}$$

5. Calculate the amount of heat produced in 10 minutes in a 40-watt lamp.

[Inter. Board. 1934.]

Heat developed

$$= \frac{ECt \times 10^7}{J} \text{ calories in } t \text{ secs.}$$

∴ Heat developed in 10 minutes.

$$= \frac{40 \times 10 \times 60 \times 10^7}{4 \cdot 2 \times 10^7} \text{ cal.}$$

(Since E × C = 40 and $J = 4 \cdot 2 \times 10^7$ ergs per calorie.]

Hence the heat produced

$$= 5714 \cdot 3 \text{ calories.}$$

6. In a hostel of 40 boys there are 48 electric lamps, each of which is marked 40 watts, 220 volts. All the lamps burn on an average for 5 hours daily. The current costs 5 annas per kilowatt-hour. What will be the cost of the current for a month of 30 days? If the cost of the current has to be paid by the students, what must each contribute ?

[Inter. Board. 1935.]

Each lamp consumes 40 watt-hours.

$\therefore$ 48 lamps will consume 40×48 watt-hours.

$$= \frac{40 \times 48}{1000} \text{ kilowatt-hours.}$$

The lamps burn on an average for 5 hours daily.

$\therefore$ Power consumed by all the lamps in 30 days

$$= \frac{40 \times 48}{1000} \times 5 \times 30 \text{ kilowatt-hours}$$

$$= 288 \text{ kilowatt-hours.}$$

The cost of 1 kilowatt-hour is 5 annas.

$\therefore$ Cost of 288 kilowatt-hours.

$$= \frac{288 \times 5}{16} \text{ Rs.}$$

$$= \text{Rs. } 90.$$

Hence the total cost of the current = Rs. 90.

Since there are 40 students in the hostel, each student will contribute Rs. $\left(\frac{90}{40}\right)$ or Rs. 2—4 as.

7. Ten 220 volt half-watt lamps are installed in a house Find out the resistance of the combination, the candle-power of each lamp is 80. Find out the number of units (kilowatt-hours) consumed in a month of thirty days, if the lamps burn for five hours per day. [Inter. Board. 1929].

The lamps being half-watt lamps the power consumed for each candle-power of light given $= \frac{1}{2}$ watt. Since each lamp is of 80 candle-power, the power consumed by each lamp $= \frac{1}{2} \times 80 = 40$ watt-hours.

$\therefore$ 10 lamps will consume $40 \times 10 = 400$ watt-hours.

Lamps burn for 5 hours per day.

$\therefore$ Total power consumed by all the lamps in 30 days

$$= 400 \times 5 \times 30 \text{ watt-hours}$$

$$= \frac{400 \times 5 \times 30}{1000} \text{ kilowatt-hours}$$

$$= 60 \text{ kilowatt-hours.}$$

Power consumed by each lamp = 40 watt-hours.

i.e. $E \times C = 40$

$$\therefore C = \frac{40}{E}$$

$$= \frac{40}{220}$$

$$= \frac{2}{11} \text{ ampere.}$$

$$\text{But } R = \frac{E}{C} \text{ (by ohm's law)}$$

$$= \frac{220}{\frac{2}{11}}$$

$$= \frac{220 \times 11}{2} = 1210 \text{ ohms.}$$

Since the lamps are connected in parallel, the resistance of the combination.

$$= \frac{1210}{10}$$

$$= 121 \text{ ohms.}$$

8. A 25 candle-power incandescent lamp takes a current of 1·1 amperes when the potential difference across its terminals is 200 volts. Find the efficiency of the lamp and the horse-power required to supply 100 of these lamps in parallel. [1. H. P. = 746 watts.]

Number of watts consumed by the lamp

$$= E \times C.$$

$$= 200 \times 1{\cdot}1$$

Efficency of a lamp is the watts consumed for each candle-power of light given. Since the lamp is of 25 candle-power, the efficiency is

$$\frac{200 \times 1\cdot 1}{25} = 8\cdot 8 \text{ watts per c. p.}$$

Number of watts consumed by 100 such lamps.

$$= 100 \times (200 \times 1\cdot 1)$$

$$= 22000.$$

$$\therefore \text{ The required H. P} = \frac{22000}{746} = 29\cdot 5 \text{ (nearly)}$$

9. Lamps aggregating 1 ohm resistance are supplied through leads of 0·02 ohm from a source at 51 volts. The voltage is subsequently raised to 255 and the lamps replaced by high-voltage lamps consuming the same total energy. Calculate the saving per 1000 hours at four pence per kilowatt-hour. (U. L.)

$$C_1 = \frac{E}{B+R} = \frac{51}{1+\cdot 02} = \frac{51}{1\cdot 02}$$

Hence the rate of consumption of energy in the lamp in 1 sec.

$$= C^2R = \left(\frac{51}{1\cdot 02}\right)^2 \times 1.$$

$$= 2500 \text{ watts.}$$

In the second case when the lamps are replaced by high-voltage lamps of resistance aggregating x ohms, the current is given by,

$$C_2 = \frac{255}{x + \cdot 02}$$

Hence the energy consumed per second

$$= \left(\frac{255}{x + \cdot 02}\right)^2 \times x = 2500 \quad \text{(energy consumed being the same).}$$

$\therefore \; x = 25\cdot 97$ ohms nearly.

Hence the difference in the rates of consumption of total energy in the lamps and leads in the two cases

$$= \left(\frac{51}{1\cdot 02}\right)^2 \times 1\cdot 02 - \left(\frac{255}{25\cdot 97 + \cdot 02}\right)^2 \times 25\cdot 99.$$

$$= \frac{51^2}{1\cdot 02} - \frac{255^2}{25\cdot 99}$$

$$= \frac{(51)^2}{53\cdot 94} \text{ watts.}$$

Hence the gain per sec is $\frac{51^2}{53\cdot 94}$ joules.

$\therefore$ The gain in 1000 hours

$$= \frac{51 \times 51 \times 60 \times 60 \times 1000}{53\cdot 94} \text{ joules.}$$

But we know that 1 kilowatt-hour

$= 1000$ watts in 1 hour

$= 1000 \times 60 \times 60$ joules per sec.

$\therefore$ The gain in kilowatt-hour.

$$\frac{51 \times 51 \times 60 \times 60 \times 1000}{53{\cdot}94 \times 1000 \times 60 \times 60}$$

Hence the required saving

$$= \frac{4 \times 51 \times 51}{53{\cdot}94}$$

$= 196$ d. nearly.

End

Exercises.

1. A current of 5 amperes flows for 3 minutes through a wire whose resistance is 2 ohms; given that 1 water-gram-degree = 4·2 joules, find the amount of heat in water-gram-degrees, generated in the wire.

(Oxford Local Senior. 1908).

[Ans. 2143 units],

2. What will be the ratio of the currents which will produce in 1 second the same amount of heat in two wires of the same material and length, if the radius of one wire is twice that of the other ? (B of E. 1902).

[Ans. 2 : 1].

3. If a cell has an E. M F. of 1·08 volts and 0·5 ohm internal resistance, and if the terminals are connected by two wires in parallel of 1 ohm and 2 ohms resistance respectively, what is the current in each and what is the ratio of the heats devolped in each ? (L. U. Inter. B. Sc. 1902).

[Ans. 0·62, ·31 (ampere).

$$\frac{H_1}{H_2} = \frac{2}{1}.$$

4. An electric battery of constant E. M. F., having an internal resistance of 5 ohms, is connected to resistance coils of 10 ohms and 20 ohms respec-

tively, arranged (1) in series (2) in parallel. Neglecting the resistance of the connecting wires, compare the amounts of heat produced in the two cases *(a)* in the whole circuit, *(b)* in the two coils. (B. of E.)

[Ans. (a) 1 : 3; (b) 1 : 2.]

5. If the heating effect in a certain resistance box endangers the constancy of the coils when the energy used in them exceeds 0·0001 watt per ohm, find the limiting safe voltage applied to the box when the resistance 1·5 ohm is being used, and also when 2500 ohms resistance is being used. (L. U.)

[Ans. 0·015 volt ; 25 volts.]

6. A current of 10 amperes is sent through a platinum wire, the resistance of which is 2 ohms. Find the mechanical equivalent in ergs of the heat generated per second. (B. of E.)

[Ans. 2×10^9 ergs per second].

7. A coil of wire of resistance 2 ohms is soldered to two thick copper rods immersed in 1000 grams of oil (specific heat 0·6). A current of strength 3 amperes is passed for 30 minutes. Neglecting the water equivalent of the calorimeter, loss of heat by radiation etc., find the rise in temperature of the oil. A current of 1 ampere passing through a resistance of 1 ohm

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for 1 second generates 0·2387 calorie (C. U. 1909).

[Ans. 12·9°C.]

8. A current of 1 ampere, flowing for 1 second through a resistance of 1 ohm, produces 0·239 gram-centigrade units of heat. What current would have to flow for an hour through a resistance of 41·84 ohms in order that the heat produced might suffice to raise a kilogram of water from 0°C. to the boiling-point? (B of E. 1896).

[Ans. $1\frac{2}{3}$ amperes.]

9. It is required to generate 10 kilograms of steam per hour with power developed from a 110-volt circuit. What resistance should the heating coil have in order to do this, supposing loss from radiation negligible? (B. of E.)

[Ans. 1·95 ohms.]

10. A stream of water flows at a uniform rate through a glass tube where it is heated by a current traversing a manganin wire. 186 gm. of water were collected in 10 min. when the steady temperature difference between the inflowing and out-flowing water was 4·35°C. Given that the current in the wire was 1·82 amperes and the potential difference between the ends 3·12 volts, calculate the number of joules equivalent to one calorie. (U. L. Inter. 1927).

[Ans. 4·21.]

11. It is observed that the temperature of 300 gm. of liquid in a vacuum flask rises 3°C per minute when a current of 1·8 amperes passes through a heating coil of 10 ohms resistance immersed in the liquid. If the specific heat of the liquid is 0·45 cal. per gm., find the water equivalent of the flask. (U. L.).

[Ans. 19·3].

12. A current of 5 amperes is passed through a wire and therein produces 500 calories per second. If the current were increased to 7 amperes, what number of grams of water would it heat in 1 hour to 100°C? Assume that the resistance of the wire does not change, and that the initial temperature of the water is 15° C. (U. L.)

[Ans 41505·8 grams].

13. A lamp, the voltage between the terminals of which is 100, is placed in a calorimeter, which is immersed in 400 grams of water; the water-equivalent of the calorimeter being 40; the temperature is found to rise, 2·5°C per minute. Find the current in the lamp.

(Camb. Local Senior 1907).

[Ans. 0·77 ampere].

14. Compare the amounts of heat developed in the four arms of a balanced Wheatstone's bridge

when the arms have the resistance 100 : 10 : 300 : 30 ohms, respectively. (Inter. Board. 1931)

[Ans. 30 : 3 : 10 : 1].

15. Calculate the amount of heat produced in 5 minutes in a 20 Watt lamp. [Inter. Board. 1928.]

[Ans. 1440 calories.]

16. Find the current required to light a 50 Watt, 200 Volt lamp, and calculate the heat produced, assuming that only 10 per cent of the electrical energy is converted into light. (Inter. Board. 1926).

[Ans. $\frac{1}{4}$ amp ; 10·8 calories]

17. Find the cost of running 40 16-candle-power lamps for 6 hours, if each lamp requires 3·6 watts per candle, and if each Board of Trade unit costs 2d.

(Ans, 2s. 4d. nearly).

18. Describe what arrangements you would make if you wanted to run a single 50-volt incandescent lamp off a 110-volt circuit. If the lamp takes 0·5 ampere, how much power is taken from the mains, and how much power is absorbed by the lamp ?

(B. of E.)

[Ans. 55 watts ; 25 watts.].

19. A University hostel has 360 lamps installed. The lamps consume 50 watts each and are lighted up

for 6 hours daily for 9 months. The voltage of the supply is 220 and the current costs 6 annas per kilowatt-hour. Find the cost of the current as well as the maximum current used. (Inter. Board. 1930)

[Ans. Rs. 10935 ; 81·82 amp.]

20. A dynamo feeds 1000 16-candle-power lamps. What current must the dynamo supply, if the difference of potential at its terminals is 200 volts, and each lamp absorbs 3·6 watts per candle ? (B of E.)

[Ans. 288 amperes.]

21. The wires leading to a group of seven glow lamps, arranged parallel, have resistance 1 ohm, and the lamps have each resistance 70 ohms. Find the ratio of the heat developed in the lamps to that developed in the leading wires. (Int. Sc. 1893.)

[Ans. 10 : 1

22. Find the power required to maintain a current of ·75 ampere in each of the following systems : (*a*) 100 lamps of 45 volts connected, each by leads of 1 ohm, in multiple arc with a dynamo ; (*b*) the same number of lamps in ten rows of ten lamps each, the leads to each row having a resistance of 10 ohms. (Camb. Sc. 1891).

[Ans. 343·25 watts in each case.]

3431·25

23. Twenty metal filament lamps, each of 32 candle-power and taking 1·4 watts per candle, are installed in a house and supplied, at 250 volts. What is the total current taken when the lamps are all lighted, and the cost per hour if electricity is supplied at four pence per Board of Trade unit? (C. G.)

[Ans. 3·58 amp ; 3·58 d.]

24. An electric light installation consists of a group of lamps in parallel arc between the ends of leads. The leads have a total resistance of 0·4 ohm, and bring current from 60 accumulators, each with E. M. F. 2 volts and resistance 0·01 ohm. When 25 lamps are switched on, each takes 0·4 ampere. Find the resistance of a lamp, and the watts used.

[Inter. Sc. (Hons.) 1894].

[Ans. 275 ohms ; 44 watts per lamp.]

25. Calculate the power required to light 80 incandescent lamps, if the E. M. F. required be 65 volts, and the current required by each, be 0·8 ampere. If the lamps be all in parallel and the leads have a resistance of 0·5 ohm, calculate the power wasted in them. (B. Sc. 1892).

(Ans. (1) 4160 watts; (2) 2048 watts.)

26. An electric lamp is marked, 30 W. 220 V. What will be the strength of the current passing through the filament? Find the cost of using this lamp for 1 hour if the price of electric power is 5 annas per kilowatt-hour.

(Ans. $\frac{3}{22}$ amp; ·15 anna)

27. The lighting of a common room of a hostel requires 300 candle-power and the lamps supplied are half-watt gas filled lamps. Calculate the cost of lighting the room for one month if the lamps are lighted up for 6 hours daily. The cost of supply is 5 annas per kilowatt-hour.

[Ans Rs. 8—7 as.]

28. A house is installed with 50 electric lamps, each of which is marked 50 watts, 220 volts. On an average the lamps burn for 6 hours daily. If the current costs 4 annas per kllowatt hour, find the cost of the current for a fortnight and the cost on each lamp.

[Ans Rs 56—4 as; Rs. 1—2 as.]

29. A University hostel is installed with 220 half-watt lamps of 80 c. p. each. The lamps are lighted up for 5 hours daily for one month. The voltage of the

supply is 110 and the current costs 3 annas per kilo-watt hour. Find the maximum current used and the cost of the current.

[Ans. Rs. 247—8 as. ; 80 amp.]

30. 80 lamps of 25 candle-power are run by a dynamo. If the efficiency of the lamps is 2 watts (per candle-power) and they are 50 volts lamps ; find the out put of the dynamo and the current taken by each lamp.

[Ans. 4000 watts (four-unit dynamo *); 1 ampere.]

31. 100 candle-power gas-filled lamp takes a current of 0·5 ampere when the voltage is 220. Calculate (1) the resistance of the hot filament (2) the efficiency of the lamp (3) the consumption of power (in watts) and (4) the cost of using the lamp for one hour if the charge for electric power is 6 annas per unit.

[Ans. 440 ohms; 1·1 watts per c. p ; 110 watts; ·66 anna.]

32. In an Intermediate college hostel of 80 boys there are 96 electric lamps, each of 30 c. p. and efficiency 1·5. The supply is at 220 volts and the

* *Note.*—The out put of a dynamo is frequently expressed in uuits of 1000 watts each : thus a four-unit dynamo produces 4000 watts.

lamps burn on an average for $2\frac{1}{2}$ hours daily. Find the cost of the current for a month if the current costs 5 as. per kilowatt-hour. What has each student to contribute towards the expense ?

[Ans. Rs. 101 —4 as ; Re. 1 – 4 as.—3 p]

33. Two hundred lamps each taking 0·45 ampere at a pressure of 200 volts are run for 24 hours. Find the H P. of the dynamo required to run the lamps. Find also the cost at 2 annas per Board of Trade unit.

[Ans. H. P = 24·13 (nearly) ; Rs. 54].

34 The lighting installation of a college hostel consists of 240 incandescent lamps each having a resistance of 65 ohms and requiring a p. d. of 200 volts. Find the rate of working in kilowatts and in horse-power required to maintain them incandescent.

[Ans. 147·7 Kilo-watts; 197·97 H. P.]

35. Two circuits whose resistances are respectively 1 ohm and 10 ohms are arranged in parallel. Compare the amount of current passing through each of these circuits with that through the battery. Compare also the amount of heat developed in the same time in the two circuits. (B. E)

[Ans. 10 : 11 ; 1 : 11 ; 10 : 1.]

36. The E. M.-F. of a battery is 18 volts and its resistance 3 ohms. The P. D. between its poles when they are joined by a wire A is 15 volts, and falls to 12 volts, when A is replaced by another wire B. Compare the the resistances of A and B, and the amounts of heat developed in them in equal times. (B. E.)

$$\left[\text{Ans.}\ \begin{array}{l}\text{(A) 15 ohms, (B) 6 ohms;}\\ \dfrac{H_A}{H_B} = \dfrac{5}{8}\end{array}\right]$$

Induced Current, induced Electromotive Force.

It was established by Faraday that an induced E. M. F. is produced in a conductor which is moved so that it cuts the lines of force of a magnetic field. It depends upon the number of lines cut per second.

If a conductor moves at such a rate that it cuts one line of magnetic force per second, one absolute unit of E. M. F. is generated.

If N lines of force cut n conductors in t seconds (the conductors being connected in series and the rate of cutting being uniform during that time) then the induced E. M.-F. is given by

$$e = \frac{N \times n}{t} \text{ absolute units.}$$

$$= \frac{N \times n}{t} \times \frac{1}{10^8} \text{ volts.}$$

$$= \frac{\text{lines cut} \times \text{turns}}{\text{time of cutting}} \cdot \frac{1}{10^8} \text{ volts.}$$

Exercises.

1. Find the E. M. F. generated in a conductor when cutting 400 magnetic lines in 2 minutes.

[Ans. $3\frac{1}{3}$ C. G. S. units].

2. Find the E. M. F. generated (in volts) in a conductor when cutting 6000 magnetic lines in 20 secs.

[Ans. 3 microvolts.]

3. In what time will an E. M. F. of 4 volts will be produced in a conductor when cutting 400 lines. ?

[Ans. 10^{-6} second.]

4. A wire cuts through a magnetic field of strength 3000 units at a uniform speed of 1000 cm. per sec. Calculate the induced voltage between the ends, if its length is 20 cm.

[Ans. 0·6 volt].

5. A coil of 100 turns and mean diameter 50 cm. rotates about a vertical axis at the rate of 20 revolutions per second at a place where H = ·36 gauss. Find the average value of E. M. F produced.

The number of lines threading the coil when its plane is at right angles to the magnetic meridian is πr_e^2 H, and these lines are cut four times during one revolution. Hence the average E. M. F is given by

$$e\ (average) = \frac{4 \times \cdot 36 \times \pi\ (25)^2 \times 100}{\frac{1}{20}}$$

$$= 5657143 \text{ absolute units.}$$

$$= 5657143 \times \frac{1}{10^8} \text{ volt}$$

$$= \cdot 0566 \text{ volt.}$$

6. A copper disc having a diameter of 40 cm. is rotated about a horizontal axis perpendicular to the disc and parallel to the magnetic meridian. Two brushes make contact with the disc, one at the centre and the other at the edge. If the value of the horizontal component of the earth's field is 0·2 C. G. S., find the potential difference in volts between the two brushes when the disc makes 3000 revolutions per minute. (B. of E. 1906).

[Ans. $12 \cdot 57 \times 10^{-5}$ volt].

7. A copper disc of 10 cm. radius spins on is axis (perpendicular to its plane) 3000 times a minute in the earth's horizontal field of force : find the E. M. F. between centre and circumference of disc. (H = 0·18). (Vict. B. Sc. 1890).

[Ans. 2827 C. G. S. units].

8. A wire in the form of a circle 10 cm. in diameter is spinning about a diameter as axis (which is placed vertically) at the rate of 10 revolutions per second in the earth's magnetic field ($H = 0{\cdot}18$). Calculate the maximum E. M. F. generated during a revolution. (Vict. B. Sc. 1894).

[Ans. 565·5 C. G. S. units or $5{\cdot}655 \times 10^{-6}$ volt].

9. Two horizontal rods are placed parallel and at a distance of 1 metre apart. A third rod slides over them parallel to itself with an uniform velocity of 10 metres per second. Find in volts the E. M. F. between the ends of the fixed rods, assuming the earth's vertical magnetic force to be 0·47 C. G. S. units. (Camb. School 1891)

[Ans. 0·00047 volt.]

10. A coil of wire consisting of 50 turns in the form of a circle 30 cm. in diameter rotates 20 times per sec. about a vertical axis. Find the average value in volts of the E. M. F. produced if the earth's horizontal magnetic force be 0·18 C. G. S. units. Explain how you obtain the result? (Int. Sc. (Hons) 1889.)

[Ans. 0·02036 volt.]

CHAPTER VII.

SOUND.

Velocity of Sound. It was proved by Newton that the velocity of Sound in any medium is given by the expression $V = \sqrt{\frac{E}{D}}$ where E is the coefficient of elasticity and D—the density of the medium. The velocity of sound as calculated by this formula does not agree with the observed value. This discrepancy between these two values remained unexplained until Laplace pointed out, that the compressions produced by a sound-wave in any gas, take place so rapidly that there is no time for the heat to escape from the portion of gas in which it is produced, and so the adiabatic elasticity must be taken in calculating the velocity. Applying this correction known as Laplace's correction we have,

$$V = \sqrt{\frac{\gamma E}{D}}.$$

Where $\gamma \left(= \frac{C_p}{C_v}\right)$ is the ratio between the specific heat of air at constant pressure and its specific

heat at constant volume. For air the adiabatic elasticity is found to be 1·41 times the isothermal elasticity.

It can be shown that the elasticity of a perfect gas is equal to its pressure provided that its temperture remains constant during the compression. Then the above formula becomes $V = \sqrt{\frac{\gamma P}{D}}$

The velocity of sound in air at $t°$C is given by

$$V_t = V_0 \sqrt{1+\alpha t} \quad .. \quad .. \quad (1).$$

where $\alpha = \frac{1}{273}$ and t is the temperature in centigrade.

Substituting the value of α we get,

$$\frac{V_t}{V_0} = \sqrt{\frac{273+t}{273}}$$

i.e. the ratio of the velocities of sound in the same gas at two different temperatures is proportional to the square roots of the absolute temperatures.

$$\text{Also } V_t = V_0 . (1+\alpha t)^{\frac{1}{2}}$$

$$= V_0 . (1+\tfrac{1}{2}\alpha t)$$ (by the Binomial Theorem, higher powers of αt have been neglected as their product is small.)

$$i.e.\ V_t = V_o + \tfrac{1}{2} V_o \alpha t$$

$$= 332 + 6t \quad \ldots \quad \ldots \quad \ldots \quad (2)$$

(by substituting $V_o = 332$, and $\alpha = \frac{1}{273}$)

Velocities in different gasses.——

Again if V_1 and V_2 are the velocities of sound at the same temperature in two different gases which have the same number of atoms to the molecule and therefore the same value of γ, we shall have

$$\frac{V_1}{V_2} = \frac{\sqrt{\gamma \frac{P}{d_1}}}{\sqrt{\gamma \frac{P}{d_2}}}$$

Where d_1 and d_2 are the respective densities,

$$= \sqrt{\frac{d_2}{d_1}}$$

i.e. the velocities are inversely as the square roots of the densities.

Vibration in pipes and resonance tubes :—

It can be proved that $v = n\lambda$, where v is the velocity, λ — the wave length of the note, and n — the number of vibrations per second.

Note :—The equation (2) is to be used when the difference of temperature is small and approximate value is required. In all other cases the equation (1) has to be used.]

In the case of a closed pipe which is sounding the fundamental note, the open end is an antinode and the closed one is the next node and hence $\lambda = 4\,l$, where l is the length of the pipe.

Hence $v = n \times 4\,l$.

In the case of an open pipe both the open ends are antinodes with a node in the middle and hence $\lambda = 2\,l$ and $\therefore\ v = n \times 2l$

If a vibrating tuning fork be held at the mouth of an open tube (immersed in a glass cylinder full of water) which is slowly raised, it is found that at a point a reinforcement of the sound is heard. At this point the period of the air column is just the same as that of the fork and $v = n\,\lambda = 4\,n\,(\,l + {\cdot}6\,r)$ where l is the length of the tube (the length of first resonance) and r is the internal radius of the tube. The quantity $\cdot 6\,r$ is called the End-correction.

In order to determine end-correction find the lengths of the first two resonance columns and calculate by the formula :—

$$\text{End-correction} = \frac{l_2 - 3l_1}{2}$$

Vibration in a stretched string :—

It can be proved that a sound-wave travels along a stretched wire or string with a velocity

$V = \sqrt{\frac{T}{m}}$ where T is the stretching force and m is the mass of the wire per unit of length.

Also $V = n\lambda$ where λ is the wave length of the note and n — the number of vibrations per second.

When a string is sounding its fundamental note then l — the length $= \frac{\lambda}{2}$ and

$$n = \frac{V}{\lambda} = \frac{V}{2l} = \frac{1}{2l}\sqrt{\frac{T}{m}}.$$

(If l is measured in cm. then T is in dynes and m is in grams).

(This is the case of transverse vibration of a string).

Doppler's principle :—

It was first pointed out by Doppler that the frequency of a note is apparently changed if the distance between the observer and the source (emitting the note) changes.

If V is the velocity of sound in air and b and c the respective velocities of the source and the observer (both moving in the same direction) then the apparent frequency of the note emitted by the source is given by $\frac{n\,(V-c)}{(V-b)}$ when n represents the actual frequency of the note emitted.

See Willow's Text book of physics p.p. 271—272.

If the motion of either the source or the observer is reversed the sign of the corresponding quantity must be changed in the above expression. If wind is also blowing with a velocity w in the same direction then the apparent frequency becomes

$$\frac{n(V-c+w)}{(V-b+w)}.$$

Solved Problems.

1. A stone is dropped into a well and 4 seconds afterwards the sound of its striking the water is heard. If the temp. of air in the well is 14°C, find the depth of the well.

We know that $V_t = V_0 + {\cdot}6\ t$

$$= 332 + {\cdot}6 \times 14$$

$$= 332 + 8{\cdot}4 = 340{\cdot}4 \text{ metres per sec.}$$

Let t be the time that the stone takes to reach the surface of water in the well.

Then s (the depth of the well) $= ut + \frac{1}{2} g t^2$

$$= 0 + \frac{1}{2} gt^2$$

(since the stone starts from rest)

∴ On the other hand the time taken by the sound to reach the man will be $(4-t)$ secs. Hence the distance traversed by the sound during this interval will be $(4-t) \times V_t$.

Hence we must have,

$$V_t\ (4-t) = s = \tfrac{1}{2} gt^2.$$

$$\text{or } (4-t)\ 34040 = \tfrac{1}{2} \times 980\ t^2$$

$$\text{or } 490\,t^2 + 34040t - 136160 = 0$$

$$\text{or } 49\,t^2 + 3404\,t - 13616 = 0$$

From which we get $t = 3{\cdot}7$ seconds.

Hence the required depth $= (4 - 3{\cdot}7) \times 340{\cdot}4$

$= 102{\cdot}12$ metres.

2. A bullet is fired from a rifle with a velocity of 500 metres per second and is heard to strike a target 5 seconds afterwards. Find the distance of the target from the marksman, the temperature of air being 14°C.

$$V_t = V_0 + {\cdot}6t$$

$$= 332 + {\cdot}6 \times 14 = 332 + 8{\cdot}4 = 340{\cdot}4$$

metres per sec.

Suppose d metres be the required distance.

Then the time taken for the bullet to reach the target

$$= \frac{d}{500} \text{ secs.}$$

Also the time taken for the sound to travel from the target to the marksman

$$= \frac{d}{340{\cdot}4} \text{ secs.}$$

∴ By the question we must have

$$\frac{d}{500} + \frac{d}{340{\cdot}4} = 5.$$

Whence $d = 202{\cdot}52 \times 5 = 1012{\cdot}6$ metres.

3. Calculate the velocity of sound in hydrogen at 10°C from the following data:—

1 litre of hydrogen = 0·0899 gram and 1 litre of air = 1·293 gms. Velocity of sound in air at 0°C = 332 metres per sec.

We know that.

$$\frac{V_{hydrogen}}{V_{air}} = \sqrt{\frac{\text{Density of air}}{\text{Density of hydrogen}}}$$

$$\frac{V_{H_2}}{332} = \sqrt{\frac{1·293}{0·0899}}$$

$\therefore V_{H_2} = 332 \times 3·79 = 1260$ metres per sec. at 0°C. Again we have

$$\frac{V_{10}}{V_0} = \sqrt{1 + \frac{1}{273} \cdot 10}$$

$$= \sqrt{\frac{283}{273}}$$

$$\text{or } V_{10} = V_0 \times \sqrt{\frac{283}{273}}$$

$$= 1260\sqrt{\frac{283}{273}}$$

$\therefore V_{10} = 1283$ metres per sec.

4. A vibrating tuning fork is held over the mouth of a tall glass jar into which water is gradually poured. It is found that when the length of the air

column is 32·4 cm. the maximum reinforcement of the sound is produced. Find the vibration number of the fork.

We know that $V = n\lambda = 4nl$ where l cm. is the length of the first resonance.

$\therefore$ By the question we have $33200 = 4n \times 32{\cdot}4$.

$\therefore n = 33200/(4 \times 32{\cdot}4) = 256{\cdot}2$.

5. The frequency of the note given by an organ pipe is 312 at 15°C. At what temperature will the frequency be 320, supposing the pipe to remain constant in length? [Inter. Board. 1930].

Since the length of the pipe is constant the wave-length remains unchanged.

We know that

$$V = n\lambda.$$

From the question we have

$$V_{15} = n_{15} \times \lambda. \text{ (at } 15°\text{C)}$$

$$\text{or } V_0 \left(1 + \frac{1}{273} \times 15\right)^{\frac{1}{2}} = 312\,\lambda. \quad \ldots\ldots(1)$$

Similarly we shall have at t°C (the required temperature)

$$V_0 \left(1 + \frac{1}{273} \times t\right)^{\frac{1}{2}} = 320\,\lambda \quad \ldots\ldots\ldots(2)$$

By dividing the equation (2) by (1) we get

$$\left[\frac{273+t}{273}\right]^{\frac{1}{2}} \times \left(\frac{273}{273+15}\right)^{\frac{1}{2}} = \frac{320\ \lambda}{312\ \lambda}$$

or $$\left(\frac{273+t}{273+15}\right)^{\frac{1}{2}} = \frac{40}{39}$$

or $$\frac{273+t}{288} = \frac{40 \times 40}{39 \times 39}$$

$$\therefore \quad t = \frac{40 \times 40}{39 \times 39} \times \frac{288}{1} - 273$$

$$= 302{\cdot}96 - 273$$

$$= 29{\cdot}96^\circ \text{ C.}$$

6. A stretched string 3 ft. long vibrates 320 times a second. What will be the rate of vibration if the string is shortened hy 4 inches ?

(Inter. Board. 1935.)

We know that the frequency is given by

$$n = \frac{1}{2l} \times \sqrt{\frac{T}{m}}$$

From the question we have

$$320 = \frac{1}{2 \times 3} \sqrt{\frac{T}{m}} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots (1)$$

and

$$n_1 = \frac{1}{2\left(3-\frac{1}{3}\right)} \sqrt{\frac{T}{m}} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots [2]$$

where n_1 is the required frequency.

Dividing equation (2) by (1) we get

$$\frac{n_1}{320} = \frac{1}{2 \times \frac{8}{3}} \sqrt{\frac{T}{m}} \times \frac{2 \times 3}{1} \times \sqrt{\frac{m}{T}}$$

$$= \frac{9}{8}$$

$$\therefore n_1 = 320 \times \frac{9}{8} = 360.$$

Hence the required frequency is 360.

7. A wire 50 cm. long vibrates 100 times a second. If the length is shortened to 30 cm. and the stretching force quadrupled, what will the frequency be ? (Pre-medical Test 1935, and Intermediate Board 1927.)

We know that

$$n = \frac{1}{2l} \sqrt{\frac{T}{m}}.$$

In the first case we shall have

$$100 = \frac{1}{2 \times 50} \sqrt{\frac{T}{m}} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (1)$$

In the second case we shall have

$$n_1 = \frac{1}{2 \times 30} \sqrt{\frac{4T}{m}} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (2)$$

where n_1 is the required frequency.

Dividing equation (2) by (1) we get

$$\frac{n_1}{100} = \frac{1}{2 \times 30} \sqrt{\frac{4T}{m}} \times \frac{2 \times 50}{1} \times \sqrt{\frac{m}{T}}$$

$$= \frac{50 \times 2}{30}$$

$$\therefore n_1 = \frac{100 \times 50 \times 2}{30} = \frac{1000}{3} = 333{\cdot}3$$

Hence the required frequency is 333·3 vibrations per sec.

8. Explain how two bridges should be placed in order to divide a wire 100 cms. long into three segments whose fundamental frequencies are in the ratio of 1 : 2 : 3. [Intermediate Board. U. P. 1927].

We know that

$$n = \frac{1}{2l}\sqrt{\frac{T}{m}}.$$

If l_1, l_2 and l_3 are the required lengths, we shall have

$$n_1 = \frac{1}{2l_1}\sqrt{\frac{T}{m}},$$

$$n_2 = \frac{1}{2l_2}\sqrt{\frac{T}{m}},$$

$$n_3 = \frac{1}{2l_3}\sqrt{\frac{T}{m}}.$$

So that

$$n_1 : n_2 : n_3 :: \frac{1}{l_1} : \frac{1}{l_2} : \frac{1}{l_3}.$$

But we have by the question

$n_1 : n_2 : n_3 :: 1 : 2 : 3$

Hence $\frac{1}{l_1} : \frac{1}{l_2} : \frac{1}{l_3} :: 1 : 2 : 3$

or $l_1 : l_2 : l_3 :: 1 : \frac{1}{2} : \frac{1}{3}$

or $l_1 : l_2 : l_3 :: 6 : 3 : 2$

Hence the required segments of the wire will be

$\frac{600}{11}$ cm., $\frac{300}{11}$ cm. and $\frac{200}{11}$ cm.

9. Calculate the velocity of sound in a gas in which two waves of lengths 1 and 1·01 metres produce 10 beats in three seconds. (U. L. 1908)

Let n_1 and n_2 be the frequencies in the two cases.

But we know that $V = n\lambda$.

Hence in the two cases we shall have

$V = n_1 \times 1$ ....(1)

$V = n_2 \times 1{\cdot}01$ ....(2)

$\therefore$ From (1) and (2) we have

$n_1 \times 1 = n_2 \times 1{\cdot}01$ ....(3)

The difference in frequencies is equal to the number of beats per sec.

Hence we have,

$$n_1 - n_2 = \frac{10}{3} \ldots\ldots\ldots\ldots\ldots\ldots\ldots (4)$$

From the equations (3) and (4) we get

$$n_1 \left[1 - \frac{1}{1{\cdot}01}\right] = \frac{10}{3}$$

$$\therefore\ n_1 = \frac{10}{3} \times \frac{1{\cdot}01}{{\cdot}01}$$

Hence from the equation (1) velocity is given by

$$V = \frac{10}{3} \times \frac{1{\cdot}01}{{\cdot}01} \times 1$$

$$= \frac{1010}{3}$$

$$= 336{\cdot}\dot{6}$$

Thus the required velocity is 336·7 metres per second.

10. A column of air and a tuning fork produce 4 beats per second when sounded together, the fork giving the lower note and the temperature of the air being 15°. When the temperature has fallen to 10° the two produce three beats per second. Find the frequency of the fork. (U. L. 1908).

Let N be the frequency of the fork and n_1 and n_2 those of the organ pipe at 15° and 10° respectively.

We have by the question.

$$n_1 - N = 4$$

and $n_2 - N = 3$.

Hence $\dfrac{n_1 - N}{n_2 - N} = \dfrac{4}{3}$.

or $\dfrac{n_1 - N - (n_2 - N)}{n_2 - N} = \dfrac{4-3}{3}$ (By dividendo)

or $\dfrac{n_1 - n_2}{n_2 - N} = \frac{1}{3}$ ....(1)

We also know that $V = n\lambda$.

or $n = \dfrac{V}{\lambda}$.

Hence $n_1 = \dfrac{V_{15}}{\lambda}$

and $n_2 = \dfrac{V_{10}}{\lambda}$ (λ being constant since the length of the pipe is constant)

$$\therefore \frac{n_1}{n_2} = \frac{V_{15}}{V_{10}} = \frac{V_0\sqrt{1+\alpha t_1}}{V_0\sqrt{1+\alpha t_2}}$$

$$= \frac{V_0\left(1 + \frac{1}{2}.\frac{1}{273} \times 15\right)}{V_0\left(1 + \frac{1}{2}\frac{1}{273} \times 10\right)}$$

$$= \frac{561}{556}.$$

By applying dividendo we get

$$\frac{n_1 - n_2}{n_2} = \frac{561-556}{556} = \frac{5}{556} \quad \dots\dots\dots(2)$$

By dividing (1) by (2) we get

$$\frac{n_2}{n_2 - \mathrm{N}} = \frac{1}{3} \times \frac{556}{5} = \frac{556}{15}.$$

But $n_2 - \mathrm{N} = 3$ (given by the question)

$$\therefore\ n_2 = 3 \times \frac{556}{15} = \frac{556}{5} = 111{\cdot}2$$

Hence $\mathrm{N} = n_2 - 3 = 111{\cdot}2 - 3 = 108{\cdot}2$.

11. Two organ pipes give 6 beats per sec. when sounded together in air at a temperature of 10°C. How many beats will they give when the temperature rises to 24°C? Velocity at 0°C is 1088 ft. per sec. [Inter. Board. 1932].

Let the lengths of the two pipes be l_1 and l_2 ft. respectively.

Change in temperature will not bring about any change in the lengths of the pipes and consequently the wave length will remain unaltered.

We know that $V = n\,\lambda$.

In the first case we shall have

$V_{10} = n_1\,\lambda_1$ where n_1 is the frequency of the first pipe at 10°C.

In an organ pipe (open at both ends) $\lambda = 2l$ where l is the length of the pipe.

Hence we shall have

$$V_{10} = n_1 \times 2l_1$$

$$\text{or } n_1 = \frac{V_{10}}{2l_1} \quad \ldots \quad \ldots \quad (1)$$

Similarly we shall have in the second case

$$n_2 = \frac{V_{10}}{2l_2} \quad \ldots \quad \ldots \quad (2)$$

where n_2 is the frequency of the second pipe at 10°C.

From equations (1) and (2) we get

$$n_1 - n_2 = V_{10}\left(\frac{1}{2l_1} - \frac{1}{2l_2}\right)$$

But by the question $n_1 - n_2 = 6$,

(the number of beats is equal to the difference of frequencies at 10°C).

$$\therefore\ 6 = V_{10}\left(\frac{1}{2l_1} - \frac{1}{2l_2}\right) \quad \ldots \quad \ldots \quad (3).$$

Similarly, if n_3 and n_4 represent the frequencies of the pipes at 24°C.,

we shall have

$$n_3 - n_4 = V_{24}\left(\frac{1}{2l_1} - \frac{1}{2l_2}\right) \quad \ldots \quad \ldots \quad (4)$$

By dividing (4) by (3) we get

$$\frac{n_3 - n_4}{6} = \frac{V_{24}}{V_{10}} = \frac{V_0\sqrt{1+\alpha.24}}{V_0\sqrt{1+\alpha.10}}$$

$$\text{or} \quad \frac{n_3 - n_4}{6} = \frac{\sqrt{1+\frac{24}{273}}}{\sqrt{1+\frac{10}{273}}}$$

$$\left(\text{by putting } \alpha = \frac{1}{273}\right)$$

$$= \sqrt{\frac{297}{283}}$$

$$\therefore n_3 - n_4 = 6\sqrt{\frac{297}{283}}$$

$$= 6{\cdot}15.$$

But $n_3 - n_4 =$ number of beats at 24° C.

Hence the required number of beats at 24°C = 6·15.

12 The specific gravities of oxygen and nitrogen gases are 16 : 14. At what temperature will the velocity of propogation of sound through oxygen be the same as that through nitrogen at 15°C ? (U. L. 1893).

Let $t^\circ C$ be the required temperature.

The velocity of sound in oxygen at $t^\circ C$ is given by

$$V_t = V_o \sqrt{1+\alpha t} \quad .. \quad ... \quad (1)$$

where V_o is the velocity of sound in oxygen at $0^\circ C$.

Similarly in the case of nitrogen gas we shall have

$$V_{15} = V_n \sqrt{1+\alpha 15} \quad .. \quad .. \quad (2)$$

where V_n represents the velocity in nitrogen at $0^\circ C$.

Dividing the equation (1) by (2) we get

$$\frac{V_t}{V_{15}} = \frac{V_o \sqrt{1+\alpha t}}{V_n \sqrt{1+15\alpha}}$$

But by the question we have

$$V_t = V_{15}$$

Hence $V_o \sqrt{1+\alpha t} = V_n \sqrt{1+15\alpha}$ ..... (3)

Again we know that

$$\frac{V_{oxygen}}{V_{nitrogen}} = \sqrt{\frac{14}{16}}$$

Hence the equation (3) becomes.

$$\sqrt{\frac{14}{16}} = \frac{\sqrt{1+15\alpha}}{\sqrt{1+\alpha t}}$$

or $\frac{14}{16} = \frac{1+15\alpha}{1+\alpha t}$

or $7+7\alpha t = 8+120\alpha$.

Putting the value of $\alpha = \frac{1}{273}$ we get

$t = 56 \cdot 1°C.$

13. If the velocity of sound in air at 0°C. is 332 metres per second, find the shortest length of a tube, open at both ends, that will be thrown into resonant vibration by a fork whose frequency is 256 when the temperature of the air is 51°C. (U. L. 1909).

We know that

$$V_t = V_o \sqrt{1+\alpha t}$$

$$\therefore V_{51} = 332 \sqrt{1+\alpha t}$$

$$= 332 \left(1 + \frac{1}{273} \times 51\right)^{\frac{1}{2}}$$

$$= 332\left(\frac{324}{273}\right)^{\frac{1}{2}}$$

$$= 361 \cdot 6 \text{ metres per sec.}$$

Again we know that

$$V = n\lambda$$

$$\therefore \lambda = \frac{V}{n}$$

$$= \frac{361 \cdot 6}{256}$$

We know that when an open tube gives its fundamental note the length of the tube $= \frac{\lambda}{2}$.

Hence the required length of the tube

$$= \frac{361 \cdot 6}{256} \times \frac{1}{2}$$

$$= \frac{11 \cdot 3}{16} \text{ metres}$$

$$= \frac{1130}{16} \text{ cm.}$$

$$= 70 \cdot 6 \text{ cm.}$$

14. Two locomotives travelling at the rate of 30 miles per hour approach each other when one sounds its whistle emitting 1000 waves per second. What will be the apparent frequency of the whistle to an observer in the other locomotive (1) before the locomotives meet (2) after they have crossed each other? Find also the apparent frequencies to a stationary observer near the line. (Velocity of sound = 1100 ft. per sec)

30 miles per hour = 44 ft. per sec.

The whistle is moving with a velocity of 44 feet per sec. and it emits 1000 waves per second. These waves on account of the motion of the source will be

contained in a length $=(1100-44)$ ft. and hence their apparent wave length $= \dfrac{1100-44}{1000}$ ft.

The observer is approaching the whistle with a velocity of 44 ft. per sec. and hence the length of the block of waves passing the observer is $(1100+44)$ ft. per sec. This gives the apparent velocity of sound.

But we know that

$$\text{apparent frequency} = \frac{\text{apparent velocity}}{\text{apparent wave length}}$$

Thus the apparent frequency

$$= (1100+44) \div \frac{(1100-44)}{1000}$$

$$= \frac{1144 \times 1000}{1056}$$

$$= 1083{\cdot}3.$$

(2) After they have crossed, the source and the observer both are receding from each other. Hence the apparent frequency is given by

$$(1100-44) \div \frac{(1100+44)}{1000}$$

$$= \frac{1056 \times 1000}{1144}$$

$$= 923{\cdot}1.$$

When the locomotive is approaching the stationary observer near the line, 1000 waves will be contained

in a length $(1100 - 44)$ ft. and hence the apparent wave length $= \dfrac{1100 - 44}{1000}$.

Hence the apparent frequency

$$= 1100 \div \frac{(1100 - 44)}{1000}.$$

$$= \frac{1100 \times 1000}{1056}$$

$$= 1041 \cdot 6.$$

When the locomotive is receding from the stationary observer, the waves will be contained in a length $(1100 + 44)$ ft.

The apparent frequency therefore becomes

$$= 1100 \div \frac{1100 + 44}{1000}$$

$$= 961 \cdot 5.$$

Exercises.

[*Note* :—The velocity of sound in air at 0°C, is 332 metres per second unless otherwise mentioned.]

1. If a bell is struck immediately at the level of the sea and its sound, reflected from the bottom, is heard 3 seconds after, what is the depth of the sea ?

[Take velocity of sound in sea-water = 4714 ft. per sec].

(Allahabad University). [Ans. 7071 feet]

2. A bell is struck near the bottom of a sea. Its sound reflected from the bottom is heard 3 secs. later. If the depth be 4290 metres and if the density of sea-water be 1·1, find out the elasticity of the sea-water.

[Ans. $2{\cdot}25 \times 10^{10}$].

3. At what distance is an observer from an echo which repeats a sound after 3 seconds, the temperature of air being 10°C ? (Patna University)

[Ans. 507 metres].

4. The report of a cannon is heard 15 seconds after the flash is seen. If the temperature of the air is 22°C, find the distance of the cannon.

(Patna University) [Ans. 5175 metres]

5. A stone is dropped into a well and is heard to strike the water 4 seconds afterwards. Find the depth of the well, given the temperature of the air in the well to be 36·4°C. [$g = 980$ cm/sec^2]

[Ans. 70·68 metres].

6. A person drops a stone in a well 144 ft. deep and hears the sound of the splash at the top. If the velocity of sound in air at that temperature is given to be 1120 ft. per sec., find the interval between the two actions. ($g = 32$ ft/sec^2.) [Ans. 3·129 secs.].

7. A bullet is fired from a rifle with a velocity of 414 metres and is heard to strike a target 4 seconds afterwards. Required the distance of the target from the marksman. the temperature being assumed to be zero. (Allahabad University.)

[Ans. 737 metres.]

8. A stone is thrown into a well with a velocity of 12 metres and is heard to strike the water 4 seconds afterwards. Find the depth of the well.

(Allahabad University.) [Ans. about 110 metres].

9. Between a flash of lightning and the moment at which the corresponding thunder is first heard, the interval is the same as that between two beats of the pulse. Knowing that the pulse makes 80 beats in a minute and assuming the temperature of the air to be 15°C, what is the distance of the discharge ?

[Allahabad University). [Ans. 255·75 metres].

10. A person stands 150 ft. on one side of the line of fire of a rifle range 450 ft. in length and at right angles to a point 150 ft. in front of the target. What is the mean velocity of the bullet if the person hears it strike the target $\frac{1}{9}$ of a second later than the report of the gun? The temperature is assumed to be 16·5°C. Velocity of sound at 0°C = 1100 ft. per sec. (Allahabad University.)

[Ans. 2046 ft.]

11. An echo repeats five syllables, each of which requires a quarter of a second to pronounce, and half a second elapses between the time the last syllable is heard and the first syllable is repeated. What is the distance of the echo, the temperature of the air being 10°C? (Allahabad University.)

[Ans 295·75 metres].

12. A man standing between two parallel cliffs fires a rifle. He hears one echo after $1\frac{1}{2}$ seconds, one after $2\frac{1}{2}$ seconds, and one after 4 seconds. Explain how these echoes reach him and calculate the distance apart of the two cliffs. The velocity of sound under the given conditions is 1120 ft. per second. (L. U.)

(Ans. 2240 feet).

13. The distance between two parallel cliffs is 560 ft. A man standing at a distance of 300 ft from one of them gives a clap. At what intervals

will he hear the echoes of the clap? The velocity of sound = 1120 ft. per second. (B A. 1926)

[Ans. $\frac{1}{14}$ second.]

14. A soldier running towards a cliff, with a uniform velocity of 4 metres per second, fires a gun at a distance of 2·49 kilometres from the cliff. When and where will he hear the echo? (Velocity of sound in air = 332 metres per sec.) (I. Sc. 1923)

[Ans. After $14\frac{23}{28}$ sec; $2430\frac{5}{7}$ metres away from the cliff.]

15. Immediately in front of you is a ship at sea firing at a target, and behind you a distant hill. Two reports of each discharge are heard, one three seconds after the other; account for this, and calculate the distance of the hill from where you are stationed.

[Velocity of sound in air, 1100 ft. per second].

(C. S. C.)

Ans. 1650 ft.]

16. An egine is approaching a tunnel surmounted by a cliff, and emits a short whistle when half a mile away. The echo reaches the engine after $4\frac{1}{2}$ seconds. Calculate the speed, of the engine, assuming the velocity of sound to be 1100 ft. per second. (L. U.).

[Ans. 50 miles per hour].

17. A steamer whistles while approaching a cliff and the echo is heard after an interval of 8 seconds. She again whistles after 5 minutes and the interval is found to be 6 seconds. How fast is the steamer going and how far is she now from the cliff ?

[Ans. 3·75 ft. per sec ; 4495 ft.]

18. A man stationed between two parallel cliffs fires a gun. He hears the first echo after 2 seconds and the next after 5 secs. What is his position between the cliffs, and when will he hear the third echo ? (Allahabad University 1919).

[Ans. 1120 ft. from one cliff and 2860 ft. from the other cliff ; 7 secs.]

19. A man walks from a high wall striking a board with a hammer once every second. How far must he go from the wall before he hears the echo of one stroke simultaneously with the next stroke.

[Oxf. Prelim. and C. (3) 08]

[Ans. $166 + \frac{x}{2}$ metres where x = speed of the man in metres per sec.]

20. Find the temperature at which the velocity of sound in air is 360 metres per second. [Ans. 48°C].

21. What must be the temperature of air in order that sound may travel in it with the same velocity as in hydrogen at 0°C ? Density of H_2 at N. T P.

$= \cdot 0899$ gms/litre ; density of air at N. T. P.

$= 1 \cdot 29$ gms/litre. (Patna University)

[Ans. $3640°C$]

22. What must be the temperature of air in order that the velocity of sound may be the same as in carbonic acid gas at $0°C$? Velocity of sound in CO_2 gas at $0°C = 2 \cdot 573 \times 10^4$ cm/sec.

(Patna University) [Ans. $-109°C$].

23. If the velocity of sound in nitrogen is 1110 ft. per second, what would be the velocity in hydrogen at the same temperature ? Density of nitrogen : density of hydrogen = 14 : 1.

(Army 1903)

[Ans. $1110 \times \sqrt{14}$ or $4153 \cdot 62$ ft/sec]

24. Find thc temperature at which the velocity of sound in air is trible the velocity of sound at $0°C$.

[Ans. 2457 absolute or 2184 °C.]

25. If sound travels with a velocity of 5200 metres per sec. in steel and if the density of steel is $7 \cdot 7$, find out Young's modulus for steel.

[Ans. $2 \cdot 082 \times 10^{12}$].

26. It was found by an experiment that sound travelled in water with a velocity of 1439 metres per

sec. at 15·1°C. Find out the elasticity of water from these data.

[Ans. $2·07 \times 10^{10}$]

27. A signal-gun is fired at a distance of 2000 metres off and an observer sets his watch by its sound. If the temperature be 15°C, find to a hundredth of a second, the error due to distance.

[Ans 5·87 secs.]

28. An observer sets his watch by the sound of a signal-gun fired at a distant cliff. He finds that his watch is 5 secs. slow. Find the distance of the observer from the cliff if the velocity of sound is 1100 ft. per second.

[Ans 5500 ft.]

29. Kendall in a North-pole Expedition, found that the velocity of sound at −40°C was 314 metres How closely does this agree with that calculated from the value we have assumed for 0°C ? (Patna Univ.)

[Ans. 7·3 metres too much].

30. Calculate the velocity of sound at 0°C, having given the following data :—

Height of barometer, 760 mm; density of mercury, 13·6 grams per c.c ; ratio of specific heats 1·41;

mass of 1 litre of dry air at 0°C, 1·29 grams; acceleration due to gravity, 981 cm. per sec per sec.

[S. K. 1894].

$V = \sqrt{\gamma h d g / D}$ where h is the barometric height and d, the density of mercury.

Substituting the given values we get

$V = \sqrt{1{\cdot}41 \times 76 \times 13{\cdot}6 \times 981 / {\cdot}00129}$

$= 33129$ cms. per sec.

31. Calculate the velocity of sound in hydrogen at -100°C. Given that at 0°C and 75 cms. of mercury 11·4 litres of hydrogen weigh 1 gram.

[Inter. Sc. 1892.]

[Ans. $1{\cdot}01 \times 10^5$ cm. per sec.]

32. Calculate the velocity of sound in water at 4°C when its coefficient of elasticity at this temperature is given to be $2{\cdot}1 \times 10^{10}$.

[Ans. 144900 cm/sec].

33. Find the velocity of sound in steel if Young's modulus of steel $= 2 \times 10^{12}$ dynes per sq. cm. and if the density of steel is 7·7 gms. per c.c.

[Ans. 5096 metres per sec.].

34. Find the velocity of sound in iron if Young's modulus of iron $= 13{\cdot}8 \times 10^{12}$ poundals per sq ft. and if 1 cubic foot of iron weighs 480 lbs.

[Ans. 17000 ft per sec.].

35. If the movable disc of a siren is perforated by 15 holes and makes 600 revolutions per minute, find the pitch and wave length of the note emitted, assuming the velocity of sound to be 340 metres per second (Army 1904).

[Ans. 150; 2·27 metres].

36. A siren having a ring of 200 holes is making 132 revolutions a minute, and it is found to emit a note which is an octave lower than that of a given tuning-fork. Find the frequnecy of the latter. (L. U.)

[Ans. 88c.]

37. If a tuning fork of frequency 256 is sounded in air in which the velocity of sound is 340 metres per second, what is the wave length of the note produced by the fork? (Army 1902).

[Ans. 132·81 cm.]

38. A glass rod is clamped at the middle and is rubbed with a wet flannel. If the length of the rod is 30 cm. long and if the frequency of the note produced is 7000, find the velocity of sound in glass.

[Ans. $4·2 \times 10^5$ cm. per sec.]

39. Water is compressed by $\frac{1}{21000}$ per unit volume by the pressure of a column of itself, 1033

centimetres high. How is this ascertained? Find from it the velocity of sound in water.

(Edinb. M. A. 1890).

[Ans, $1·459 \times 10^5$ cm. per sec.]

40. A wire is stretched by a weight of 20 kilograms and 50 cm of it gives out a certain note. If 1·5 metres of the wire is found to weigh 2·874 gm. Find the pitch of the note emitted by the wire.

[Ans. 320].

41. A wire of length 140 cm. and mass 52 grams is stretched by means of a load of 16 kilograms. Calculate the frequency of the fundamental vibration. g = 981 cm. per sec. per sec. (L. U.)

[Ans. 23·26].

42. A copper wire (density 8·8 grams per c.c.) one metre long and 1·8 mm. in diameter is stretched by a weight of 20 kilograms. Calculate the frequency of the fundamental note.

[Inter. Board. 1920; C. U. 1922].

[Ans. 46·7 vibrations per sec.]

43. A string gives out a note whose frequency is 512. If the weight by which it is stretched is 10 lbs, find the weight required to produce the note whose frequency is 384.

[Ans. 5·625 lbs].

44. The string of a certain monochord vibrates 100 times a second. Its length is doubled and its tension altered until it makes 150 vibrations a second. What is the relation of the new tension to the original? (C. U. 1924.)

[Ans 9 : 1]

45. Two wires, equal in length, and made of the same material, are subjected to tensions in the proportion 1 : 3, the one under the greater tension being the thicker. The thinner wire emits a fundamental note of double the frequency of the other. Compare the diameters of the wires, supposing them to be circular in cross-section. (L. U.)

[Ans. $1 : 2\sqrt{3}$]

46. A brass wire 1 metre in length is stretched by a weight of 2 kilograms and a silver wire of the same diameter but 3·165 metres in length, give the same number of vibrations. What is the stretching weight in the latter case? (Allahabad University).

$$\begin{bmatrix}\text{Density of brass} = 8{\cdot}4 \text{ grams per c. c.} \\ \text{Density of silver} = 10{\cdot}5 \text{ grams per c. c.}\end{bmatrix}$$

[Ans. T = 25 kilograms].

47. The density of iron being 7·8 and that of copper 8·8, what must be the thickness of wires of these materials of the same length and equally

stretched, so that they may give the same note.

(Allahabad University).

[Ans. $\frac{r_1}{r_2}$ = 1·062 where r_1 and r_2. are the radii.]

48. Find the time of the slowest vibrations of a pianoforte wire weighing 0·002 of a pound per foot, stretched between two points 5 feet apart, with a tension of 100 lbs. weight ? [Edinb. M. A. 1890.]

[Ans. 0·007905 sec].

49. A wire one metre in length is stretched so that it vibrates 80 times per second. If it weighs 160 grams find the stretching force in dynes.

[Ans. 4·096 × 10^8 dynes].

50. A brass and a silver wire of the same diameter are stretched by the weights of 2 and 2·5 kilograms respectively and produce the same note. What are their lengths, the density of brass being 8·39 and of silver 10·47? (Allahabad University).

[Ans. The length of the silver wire is 3·165 times that of the brass.]

51. The diametres of two wires of the same length and material are 0·0015 and 0·0038 mm. and their stretching weights 400 and 1600 grams respectively. Required the ratio of the numbers of their vibrations. (Patna University).

[Ans. $n_1 : n_2 = 1·267$).

52. A brass wire, density 8·5, radius 0·02 cm. is stretched between two clamps 90 cm. apart while subjected to an extension of 0·05 cm. per metre. Find the pitch of the lowest note due to transversal vibrations, assuming Young's modulus for the wire to be $9{\cdot}8 \times 10^{11}$ dynes per cm.^{2}. (Madras University).

[Ans. 42·2.]

53. A wire 50 cm. long and of mass 6·5 gms. is stretched so that it makes eighty vibrations per sec. Find the stretching force. (Inter. Board. 1925].

[Ans. 8·49 kilograms.]

54. If a string is sounding a given note, in what ratio must the tension be increased to alter the frequency in the ratio 5 : 2 ? How much would the string have to be shortened to make the same change without altering the tension ? (L. U.)

[Ans. 25 : 4 ; 2 : 5.]

55. A stretched wire under tension of 1 kilogram weight is in unison with a tuning fork of frequency 320. What alteration in the tension would make it vibrate in unison with a fork of frequency 256 ?

[Inter. Board. 1929.]

[Ans $\frac{16}{25}$ Kilogram.]

56. A sonometer string is stretched with a force 200 grams ofweight. (*a*) The force is increased to

800 grams. (*b*) The length of the string halved. How is the pitch of the note emitted by the string affected in each case? (Calcutta University).

[Ans. In both cases the first higher octave is heard].

57. A certain length of an inextensible string vibrates as a whole when stretched by a weight of 100 lbs. With what weight must double that length be stretched in order that it may vibrate at the same rate? (Prel. Sc. 1891).

[Ans: 400 lbs. weight].

58. Three strings, A, B and C, of the same length are stretched on a sonometer. Their relative weights are A : B : C : : 2 : 8 : 18, and the tensions in them are A : B : C : : 12 : 12 : 27. Calculate the ratios of their rates of vibration.

[Prel. Sc. 1889].

[Ans. 2 : 1 : 1].

59. A violin string 1 ft. 6 ins. long, when bowed transversely emits a note of pitch 256. If it weighs an ounce to the yard, determine the force which stretches it. (Army 1905).

[Ans. 384 lbs.].

60. Compare the frequencies of vibration of two strings stretched with weights of 10 kilograms and

1 kilogram respectively. They are each 1 metre long, and they are of the same diameter, but their densites are 7·8 to 1 respectively.

(Lond. Inter. Sc. 1902).

[Ans. 1·13 : 1]

61. Two similar wires of the same length are stretched — the one by a weight of 2 kilograms and the other by a weight of 4·5 kilograms. Find out the interval between the notes produced by them.

[Ans. (Fifth) *c* : *g* or 2 : 3]

62. A wire is stretched by a weight of 13 lbs. and sounds a certain note. What must be the stretching weight to produce the major third ?

(Patna University)

[Ans. Let T be the required weight. The major third is $\frac{5}{4}$ the number of vibrations of the fundamental note. The number of vibrations is directly as the square root of the stretching weight. We shall get T = 20·313 lbs.]

63. A string stretched with a weight of 25 lbs., when made to vibrate transversely, gives a certain note. What tension must be applied to a string of the same material, but of twice the length and thickness, to make it give the octave above the note ? (U. L.)

[Ans. 1600 lbs.]

64. Calculate the frequency of vibration of a closed pipe 11 ft. long. (Velocity of sound in air = 1100 ft. per sec.)

(Lond. Inter. Sc. 1906) [Ans. 25].

65. What will be the length of a closed pipe if the lowest note has a frequency of 288 at 30°C?

[Ans. 30·35 cm.]

66. The frequency of the note given by an organ pipe is 312 at 15°C. At what temperature will the frequency be 320, supposing the pipe to remain constant in length? (Inter. Board. 1930).

[Ans. 29·9°C].

67. A sounding organ-pipe is warmed by means of boiling oil from 16° to 127° centigrade. What is the effect on the note which it emits?

(Inter. Sc. 1891).

[Ans. The frequency will be increased in the ratio of 20 : 17.]

68. Calculate the change of pitch of an open organ pipe 3 feet long when the temperature changes from 10°C to 15°C. (L. U).

[Ans. 1·009].

69. The velocity of sound in hydrogen is 1926·5 metres per second. What will be the length

of a closed organ pipe filled with hydrogen which gives a note having a vibration frequency of 512 per second? (C. U. 1915)

[Ans. 63·3 cms].

70. What must be the ratio of the lengths of a closed and an open pipe in order that the third overtone of the open pipe shall be in unison with the second overtone of the closed pipe? (U. L.)

[Ans. 5 : 8].

71. An organ pipe having been tuned in the morning to give a note of frequency 256 at 15°C. is found in the evening when blown strongly to give a note of 516. Find the temperature in the second case, and state the type of the organ pipe. (L. U.)

[Ans. 19·4°C.; open pipe].

72. An organ pipe gives out a note whose frequency is 320 at 30°C. Supposing the pipe to remain unchanged in length find out the temperature when the frequency is 312.

[Ans. 15°C.]

73. A whistle gives a certain note when sounded, the temperature being 18°C. To what must the temperature be raised for it to give a note of $\frac{9}{8}$th. frequency of that at 18°C? (L. U.)

[Ans. 95·3°C.]

74. The first two air columns which give resonance with a fork are 32 cm. and 99·9 cm. respectively. Calculate the end correction.

[Ans. 1·95 cm.]

75. A tuning fork produces resonance in a glass tube where the length of the air column is 34 cm. When the air column is 105·6 cm. it also gives resonance with the same fork. Find out the end correction and the internal radius of the tube.

[Ans. End correction = 1·8 cm;
radius = 3 cm.]

76. The length of a column of air in a tube closed at one end which gives the greatest resonance to a tuning fork is observed to be 32·5 cm. Find the wave-length of the note emitted by the fork.

(Inter. Board. 1919). [Ans. $\lambda = 130$ cm.]

77. Calculate approximately the length of the resonance box closed at one end on which a tuning fork is to be mounted, the pitch of which is 256, the velocity of sound in air being 1120 ft. per sec. Would the same resonance box answer for a fork of another pitch ? If so, of what pitch [Inter. Board. 1926].

[Ans. Required length = $\frac{35}{32}$ ft. Frequency of forks are 768 ; 1280 ; 1792 and so on.]

78. A resonance tube, 2 cm. in diameter and containing water, gives the loudest note with a fork of frequency 512 when the air column in the tube above the water is 15·5 cm. long. Find (*a*) the wave length of this note, (*b*) the velocity of sound in air. (Sen. Camb. Loc.)

[Ans 64·4 cm. 329·8 metres per sec.]

79. Find the lengths of the tubes at which the air column in a resonance tube at a temperature 20°C will resound to a note of frequency 256. The velocity of sound at 0°C is 330 metres per second, and the effect of the width of the tube is neglected.

[Inter. Board. 1929].

[Ans. (1) 33·4 cm. (2) 3 × 33·4 = 100·2 cm. (3) 5 × 33·4 (4) 7 × 33·4 and so on].

80. The sound of an excited fork swells out loudly, when held over a gas jar 6·4 inches long and of 1 in. radius. Find the wave-length in air of the note emitted. Calculate also the vibration frequency of the fork. Temp. is 20°C. (Allahabad University 1919).

[Ans. 27·8″; 486].

81. The points of prongs of a tuning fork A originally in unision with a fork B of frequency 512 are filed, and the forks produce 5 beats per second when sounded together. What is the pitch of A after filing? (Inter. Board. 1916).

[Ans. 517].

82. Beats are heard three times per second when two open organ pipes one 2 feet 9 inches in length and the other half an inch longer are sounded together. Find the velocity of sound in the air at the time of the observation. (L. U.)

[Ans. 1105·5 ft. per sec.]

83. Two tuning-forks when sounded together give 3 beats per sec. One is in unison with a length of 58 cm. of a monochord string under constant tension, and the other with 59 cm. of the same string. What are the frequencies of the forks?

(Cal. University 1927). [Ans. ~~174~~ ; ~~177~~].
177 174

84. An air column and a tuning fork produce 9 beats when sounded together. The fork gives the lower note. They produce 18 beats when the temperature becomes 88°C. If the temperature in the previous case is 16°C, find out the frequency of the fork.

[Ans. 67·5].

85. A column of air and a tuning fork produce 4 beats per second when sounding together, the fork giving the lower note and the temperature of the air being 15°C. When the temperature has fallen to 10°C. the two produce 3 beats per second. Find the the frequency of the fork. (L. U.)

[Ans. 110·5] 108·2

86. Two organ pipes give 12 beats when sounded together in air at 27°C. How many will they give at 77°C?

[Ans. 12·96].

87. If two organ pipes give four beats per sec. when sounded together in air at 15°C, how many will they give in air at 0°? (U L.)

[Ans. 3·89 per sec]

88. An open organ pipe produces 8 beats per second when sounded with a tuning fork of 256 vibrations per second, the fork giving the lower note. How much must the length of the pipe be altered to bring it into accord with the fork? Velocity of sound is equal to 1100 ft. per sec. (S. K. 1894).

$\left[\text{Ans. } \frac{25}{32} \text{ inch. (increase)}\right]$

89. Two open pipes without flange, having both a length of 4 m. and diametres 12 cm. and 24 cm. respectively, are observed to give a number of beats when sounded together. Determine which pipe is to be shortened and by how much if the are to be brought into unison. Velocity of sound $= 340$ m. per sec. and correction $= 0·5\, r$, when r is the radius of cross-section of the pipe. (Madras University).

[Ans. Wide pipe to be shortened by 6 cm.]

90. Two strings vibrating transversely emit fundamental notes of frequencies 300 and 302 per second respectively. How many beats per second are produced (1) by the fundamental notes, and (2) by their first overtones? (L. U.)

[Ans. (1) 2; (2) 4.]

91. An air column at 27°C produces 10 beats with a tuning fork of frequency 480, the frequency of fork being greater than that of the air column. If the number of beats becomes trible find out the temperatures of the air column.

[Ans. 80·2°C; 2°C.]

92. Two tuning forks make 4 beats per second when sounded simultaneously. One fork makes 256 vibrations per second and the beats cease when the other fork is loaded with a small piece of wax. What is the frequency of the second fork? (L. U.)

[Ans. 260].

93. If the frequency of the whistle is 256 vibrations per second and the velocity of the engine is $\frac{1}{20}$ of that of sound, what will be the frequencies of the notes heard by the man before and after the engine passes him? (Camb. Loc. Sen. 1906).

[Ans. 269·5; 243·8].

94. A locomotive whistle emitting 2000 waves per sec. is moving towards you at the rate of 60 miles an hour, on a day when the thermometer stands at $15°C$. Calculate the apparent pitch of the whistle, and explain precisely why it is not the true pitch.

[S. K. 1891]

(Velocity of sound in air at $0°C$ = 1179 ft).

[Ans. Apparent frequency = 2157].

95. Two tuning-forks whose frequencies are approximately in the ratio 2 : 1 are producing Lissajou's, figures and it is observed that the same figure is repeated after 10 secs. On loading slightly the fork of upper pitch the figure goes through a cycle of change in 8 secs. If the fork of lower pitch has a frequency of 400, what is the frequency of the other fork before and after loading.

[Ans 799·9 ; $799\frac{7}{8}$]

96. Two tuning forks are employed to produce Lissajou's figures and it is observed that the figure 8—shaped occurs at intervals of 4 sces. If the frequency of one of the forks is 300 per second, find the possible frequencies of the other.

[Ans. 150·125; $149\frac{7}{8}$; 600·5 ; 599·5]

End